G-SEC WORKING PAPER No.26

Fiscal Sustainability of Japan:

A Dynamic Stochastic General Equilibrium Approach

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September 2009

Abstract

The purpose of this paper is to investigate the fiscal sustainability of Japan by applying a dynamic stochastic general equilibrium (DSGE) model to the Japanese economy. By introducing intermediation costs into the model, we succeed in explaining the observed relationship between the interest and GDP growth rates, which is crucial in testing for sustainability. When the projected real growth rate is 2.5 percent, the average real interest rate becomes 2.57 percent, and the debt-to-GDP ratio gradually increases stochastically so that government debt is not sustainable. To recover sustainability, the primary surplus must be 0.2 percent of GDP.

Key Words: Fiscal Sustainability, Intermediation Cost, DSGE Model

JEL Classification Numbers: H68, G12 and E62.

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1. Introduction

The huge budget deficit in recent years has raised great concerns about the long-run sustainability of Japanese fiscal stance. Since the beginning of the 1990s, government debt has accumulated so much faster than GDP growth that the outstanding stock of debt as a proportion of GDP amounts to about 1.7 as of 2006. This figure is by far the highest among OECD countries (other countries whose debt-to-GDP ratio exceeds unity are only Belgium and Greece). It is obvious that the Japanese budget is in a deficit crisis. The government, responding to a possible fiscal crisis, proposed a fiscal reform plan entitled “Basic Policies for Economic and Fiscal Policy Management and Structural Reform” (*Honebuto no Houshinn* in Japanese) in 2006 and released a plan under which it will attain a primary surplus in fiscal year 2011.

At the same time, there is the puzzling fact that the real interest rates on government bonds have remained relatively low and have not reacted sensitively to the recent increase in the debt-to-GDP ratio. Figure 1 depicts the nominal interest rate of government bond and the nominal GDP growth rate for the period 1981–2005. The interest rate was 5.0 percent on average over the period prior to the zero-interest-rate policy (i.e., 1981–1998), while the growth rate was 4.2 percent on average over the same period, with a gap of 0.8 percent[1], which is far smaller than predicted by the neoclassical growth theory. Small gaps between interest rates on government bonds and the GDP growth rate are not specific to Japan. In fact, the average realized real rates of return on government bonds in major OECD countries over the past 30 years have been smaller than the real growth rate (e.g., Blanchard and Weil, 2001). In investigating the test for fiscal sustainability, it is of extreme importance to understand the relationship between the two rates[2].

There are two strands of approaches to explaining low interest rates for government bonds from the viewpoint of the neoclassical growth theory. One approach is to introduce uncertainty into the framework of a complete, frictionless economy. Abel et al. (1989) and Bohn (1995) provide examples of stochastic growth models in which risk premium drives down the safe interest rate, often below the economic growth rate. Their argument, along with that of Zilcha (1992), demonstrates that dynamic efficiency is preserved and that the no-Ponzi condition for the intertemporal government budget constraint holds even if the safe interest rate is below the growth rate. Another approach deviates from complete markets and assumes the presence of financial intermediation costs (e.g., Bohn, 1999). Aiyagari and Gertler (1991) and Heaton and Lucas (1996) introduce intermediation costs to resolve the Mehra–Prescott puzzle[3].

In this paper, we attempt to resolve the issue of low interest rates by extending Bohn's (1999) model with financial intermediation costs to a stochastic environment. Specifically, we provide a dynamic stochastic general equilibrium (DSGE) model of an exchange economy featured by intermediation costs and heterogeneous agents. Intermediation costs reduce the deposit interest rate and hence the rate of return of the government bond, a perfect substitute for deposits, and contribute to explaining the relatively low interest rate of government bond. We use this model to investigate whether the Japanese economy can avoid a possible fiscal crisis if the released reform plan to attain a primary surplus in fiscal year 2011 is realized.

We did not take the approach on risk aversion. As Mehra and Prescott (1985) and Weil (1989) argue, when the degree of relative risk aversion takes admissible values of greater than 0.5 but less than 2, the magnitude of risk premium is at most 0.06–0.4 percent, which cannot account for the relatively low interest rate of government bond. Given that the degree of relative risk aversion reported by existing studies on the Japanese economy lies between 0.5 and 1, however, this approach does not appear to improve our results greatly. We think it essential to introduce financial intermediation costs to match the data with the model.

Our results are summarized as follows. When the targeted growth rate is 2.5 percent, the average interest rate becomes 2.57 percent in the presence of significant financial intermediation costs. We report a small gap between the interest and growth rates, which is consistent with the actual data. If the targeted primary surplus is zero on average, the debt-to-GDP ratio gradually increases from the initial level of 1.72 to 1.896 in 100 years, suggesting that the government debt is not sustainable. Sustainability requires the primary surplus to be no less than 0.2 percent of GDP. Lowering the GDP growth rate to 2 percent, 1 percent and 0 percent and assuming that the government runs a zero primary surplus for each growth rate, we find that the average interest rate of government bonds decreases but the gap between the interest rate and growth rates widens. Therefore, the government faces more difficulty in meeting the sustainability condition.

This paper belongs to the literature on methodology to test sustainability of fiscal deficits. Hamilton and Flavin (1986) tested sustainability by examining the transversality condition in the intertemporal budget constraint of the government. Bohn (1998) proposed a simple and pragmatic test for sustainability under which it is sufficient to ascertain whether the debt-to-GDP ratio displays a mean-reversion property. In their famous paper entitled "Deficit Gamble," Ball et al. (1998) projected future growth rates and interest rates using past data and calculated the probability under which the debt-to-GDP ratio enters a dangerous zone. What is common in the existing literature is that all studies rely heavily on past data, and so their approaches are of limited use for estimating the effect of possible fiscal reforms in periods of fiscal crisis.

This paper is also related to the empirical literature investigating fiscal sustainability in Japan. Doi and Ihori (2003) take the intertemporal approach following Hamilton and Flavin (1986) and reach a pessimistic conclusion regarding the period of 1965–2000[4]. A substantial body of literature examines the Bohn's condition (for example, Ihori et al., 2007) and reports findings against that condition when the sample includes the period since 1990.

Ihori et al. (2007), the study most similar to ours, simulates fiscal sustainability using a frictionless growing economy of overlapping generations, and makes predictions more pessimistic than ours. The primary reason behind their pessimistic results lies in the gap between the interest and GDP growth rates that amounts to about 3 percentage points that are greater than actual data and our study. Dekle (2005) simulates fiscal sustainability using a small-open economy version of the growth model and makes a pessimistic projection based on his assumed wide gap between the interest and TFP growth rates. Broda and Weinstein (2005) and Doi (2006) provide some fiscal implications by applying several exogenous pairs of economic growth rates and interest rates[5].

This paper is organized as follows. Section 2 introduces the model used in calibration. Section 3 reports the results of this fiscal calibration. Section 4 concludes.

2. Model

We construct a dynamic stochastic general equilibrium (DSGE) model of an exchange economy with infinitely lived agents to investigate the sustainability of government debt. We assume intermediation costs and heterogeneous agents to provide fiscal implications of low safe interest rates[6]. The developed model is a stochastic version of Bohn (1999).

The model has an equal number of two types of infinitely lived agents, A and B. Type A has income $Y_t^A = Y_t$ in even periods (zero otherwise) and type B has income $Y_t^B = Y_t$ in odd periods (zero otherwise). The growth rate of income, $x_{t+1} \equiv (Y_{t+1}/Y_t) - 1$, is a random variable that follows a Markov process according to a continuous and differentiable probability distribution function $G(x_{t+1}|x_t)$ with a positive density $g(.)$ over the support $[\underline{x}, \bar{x}]$, with a mean μ and $\bar{x} > -1$.

To simplify the notation, let there be one representative agent of each type so that individual income Y_t denotes the aggregate income. There is no population growth. The model developed here is an exchange economy without production so that x_{t+1} may be interpreted as the sum of the growth rate of per capita income and the population growth rate. Both types have preferences over consumption $\sum_{t=0}^{\infty} \beta^t u(C_t)$, where $u(.)$ is strictly increasing and concave and $0<\beta<1$. Government spending is $G_t = gY_t$, with $0 < g < 1$. The government finances its spending by imposing lump-sum taxes T_t on the type that receives income, and by issuing debt.

There is a room for private lending and borrowing among agents. We introduce market imperfections by supposing that private lenders must bear a proportional intermediation cost $\kappa > 0$ per unit of deposit. One may interpret κ as the cost of monitoring or identifying a borrower or verifying credit. The cost is measured in terms of loss of effort so that the good market is not influenced by the intermediation cost. The bank is compensated for the loss of effort by income, but the income accruing to the bank is transferred to households in a lump sum fashion. This assumption is an artifact to avoid complicating the model despite the presence of the intermediation cost. In a world of competitive intermediation, if a loan is made at the borrowing rate R_t^*, the after-cost return to the lender is $1+R_t = (1+R_t^*)/(1+\kappa)$. The key assumption of the model is that loans to the government can be monitored without cost, so that the government can borrow at the lending rate R_t.

It is useful to examine first an economy with no government. Let $V(B_{t-1}, Y_t, x_t)$ denote the value function of the agent who inherits the debt B_{t-1} from period $t-1$ and receives income Y_t in period t, and let $\tilde{V}(W_{t-1}, Y_t, x_t)$ denote the value function of the agent who held the deposit W_{t-1} in period $t-1$ and does not receive income in period t[7]. The agent who receives income Y_t in period t maximizes $V(B_{t-1}, Y_t, x_t) = \max_{C_t, W_t} u(C_t) + \beta E_t \tilde{V}(W_t, Y_{t+1}, x_{t+1})$[8] subject to the budget constraint $Y_t - B_{t-1}(1+R_t^*) = C_t + W_t$, where C_t is consumption when agents receive income. For later reference, let $\tilde{C}_t$ denote consumption when agents do not receive income. The F.O.C. for consumption yields

$$u'(C_t) = \beta E_t \, \tilde{V}_1(W_t, Y_{t+1}, x_{t+1}) \,. \tag{1}$$

Using the envelope theorem, we obtain

$$V_1(B_{t-1}, Y_t, x_t) = -u'(C_t)\,(1+R_t^*) \,. \tag{2}$$

On the other hand, the agent who receives income in period $t-1$ maximizes $\tilde{V}(W_{t-1}, Y_t, x_t) = \max_{\tilde{C}_t, B_t} u(\tilde{C}_t) + \beta E_t \, V(B_t, Y_{t+1}, x_{t+1})$ subject to the budget constraint $W_{t-1}(1+R_t) + B_t = \tilde{C}_t$. The F.O.C. for consumption yields

$$u'(\tilde{C}_t) + \beta E_t \; V_1(B_t, Y_{t+1}, x_{t+1}) = 0 \,. \tag{3}$$

Using the envelope theorem, we obtain

$$\tilde{V}_1(W_{t-1}, Y_t, x_t) = u'(\tilde{C}_t)\,(1+R_t) \,. \tag{4}$$

It follows from (1) and (4) that we derive the Euler equation for the agent who receives income in period t

$$u'(C_t) = \beta E_t \,\{u'(\tilde{C}_{t+1})\,(1+R_{t+1})\} \,. \tag{5}$$

It follows from (2) and (3) that we derive the Euler equation for the agent who receives income in period $t-1$

$$u'(\tilde{C}_t) = \beta E_t \{u'(C_{t+1})\ (1+R^*_{t+1})\}\ . \qquad (6)$$

Hereafter we proceed with the analysis by assuming that agents have identical CRRA utility functions, $u(C_t) = C_t^{1-\alpha} \big/ (1-\alpha)$, where α is the degree of the relative risk aversion and hence the inverse of the elasticity of substitution of consumption between periods. Accordingly, (5) and (6) are rewritten, respectively, as

$$1 = E_t\{\beta \frac{\tilde{C}_{t+1}^{-\alpha}}{C_t^{-\alpha}}(1+R_{t+1})\}\ , \qquad (7)$$

and

$$1 = E_t\{\beta \frac{C_{t+1}^{-\alpha}}{\tilde{C}_t^{-\alpha}}(1+R^*_{t+1})\}\ . \qquad (8)$$

In addition, market clearings in the good market are $C_t + \tilde{C}_t = Y_t$, and $C_{t+1} + \tilde{C}_{t+1} = Y_{t+1} = (1+x_{t+1})Y_t$. Let $C_t \big/ \tilde{C}_t \equiv \theta_t$ denote the consumption ratio between when agents receive income and do not. By limiting focus on an equilibrium in which θ_t is constant through time ($\theta_t = \theta_{t+1} = \theta$), we obtain

$$C_{t+1} \big/ C_t = \tilde{C}_{t+1} \big/ \tilde{C}_t = 1 + x_{t+1}\ . \qquad (9)$$

Equation (9) implies that the growth rate of consumption when agents receive income is equal to that when not, and to the growth rate of income. We use (9) to rewrite (7) as

$$1 = E_t\{\beta \frac{\tilde{C}_{t+1}^{-\alpha}}{\tilde{C}_t^{-\alpha}} \frac{\tilde{C}_t^{-\alpha}}{C_t^{-\alpha}}(1+R_{t+1})\} = E_t\{\beta(1+x_{t+1})^{-\alpha}\theta^{\alpha}(1+R_{t+1})\}\ , \qquad (10)$$

and, using (8), have

$$\frac{C_t^{-\alpha}}{\tilde{C}_t^{-\alpha}} = \frac{E_t\{\beta\tilde{C}_{t+1}^{-\alpha}(1+R_{t+1})\}}{E_t\{\beta C_{t+1}^{-\alpha}(1+R_{t+1}^{*})\}} = \frac{E_t\{\beta\tilde{C}_{t+1}^{-\alpha}(1+R_{t+1})\}}{E_t\{\beta(\theta C_{t+1}^{-\alpha})(1+\kappa)(1+R_{t+1})\}}.$$

Finally, we have

$$\theta = (1+\kappa)^{1/2\alpha} > 1 \tag{11}$$

for any $\kappa > 0$. In the presence of intermediation cost, agents fail to smooth consumption across periods. They consume more when they receive income than when they do not.

We are now ready to examine the relation between the economic growth rate and the deposit interest rate. If there were no uncertainty in the model, (10) would be reduced to $1 = \beta(1+x_{t+1})^{-\alpha}\theta^{\alpha}(1+R_{t+1})$. It follows from this equality and (11) that

$$1+R_{t+1} = \frac{(1+x_{t+1})^{\alpha}}{\beta\sqrt{1+\kappa}}. \tag{12}$$

The deposit interest rate is higher than the growth rate in the neoclassical case of $\kappa = 0$. However, if κ is positive, the deposit interest rate tends to be lower, and if κ is sufficiently high, it may become lower than the growth rate. In any case, there exists a causal relationship between them so that formulation of their relationship is crucial in investigating the test for fiscal sustainability.

Let us return to the analysis in a world with uncertainty. We assume $E_t\{\beta(1+x_{t+1})^{1-\alpha}\} < 1$ to guarantee the finiteness of the expected utility. Consider an auxiliary variable v_{t+1} that is introduced to measure the difference between R_{t+1} and x_{t+1}, to satisfy $1+R_{t+1} = (1+x_{t+1})(1+v_{t+1})$. It follows from (10) that $1 = E_t\{\beta(1+x_{t+1})^{1-\alpha}\theta^{\alpha}\}(1+v_{t+1}) < \theta^{\alpha}(1+v_{t+1})$. When there is an intermediation cost,

$(\theta > 1), v_{t+1} < 0$ is feasible, so that the deposit interest rate may be lower than the growth rate if κ is sufficiently high. Of course, for $\kappa = 0$ $(\theta = 1)$, v_{t+1} is positive; hence the deposit interest rate is always higher than the growth rate. We log-linearize (10) around the steady state and obtain

$$R_{t+1} = R + \alpha\{\log(1+x_{t+1}) - \log(1+\mu)\}, \tag{13}$$

with $R = (1+\mu)^{\alpha} / (\beta\sqrt{1+\kappa}) - 1$. The deposit is a risky asset for which the rate of return is positively dependent on the growth rate. The sensitivity of the deposit interest rate to the growth rate is dependent on the degree of relative risk aversion α.

Abel et al. (1989) and Bohn (1995) point out that the interest rate of a safe asset may be lower than the economic growth rate in the presence of uncertainty. However, as Mehra and Prescott (1985) and Weil (1989) argue, when α takes admissible values of more than 0.5 but less than 2, the magnitude of risk premium is at most 0.06–0.4 percent. Weil (1989) attempted to resolve the risk premium puzzle by proposing a preference in which the degree of relative risk aversion is unrelated to the elasticity of intertemporal substitution of consumption, but reported that the safe interest rate exceeds the actually observed value for admissible parameter values. The findings in the existing literature might suggest that intermediation cost may play an important role of explaining the lower interest rate of government bonds relative to the growth rate.

We are now ready to introduce a government bond in the model. Rigorous analysis may require an investigation of both the case when deposits and government bond are both held and the case when only the government bond is held. Bohn (1999) studies both cases in a deterministic economy, and demonstrates that if the debt-to-income ratio D_t/Y_t is below a certain threshold both assets are held, while otherwise only the government bond is held (Proposition 2 in Bohn 1999). Because the latter case is not practically feasible, we focus mainly on the interesting and realistic case when both assets are held. Later in this section, we discuss another case.

Let $V(B_{t-1}, D_{t-1}, Y_t, x_t)$ denote the value function of the agent who inherits debt B_{t-1} from period $t-1$ and the government bond D_{t-1}, and receives income Y_t in period t and let $\tilde{V}(W_{t-1}, D_{t-1}, Y_{t,}, x_t)$ denote the value function of the agent who held the deposit W_{t-1} and the government bond D_{t-1} in period $t-1$ and does not receive income in period t.

Agents who receive income Y_t in period t maximize

$V(B_{t-1}, D_{t-1}, Y_t, x_t) = \max_{W_t, D_t, C_t} u(C_t) + \beta E_t \tilde{V}(W_t, D_t, Y_{t+1,}, x_{t+1})$, subject to the budget constraint

$Y_t + (1+r_t)D_{t-1} - B_{t-1}(1+R_t^*) = C_t + W_t + D_t + T_t$. On the other hand, agents who do not receive income in period t maximize $\tilde{V}(W_{t-1}, D_{t-1}, Y_t, x_t) = \max_{B_t, D_t, \tilde{C}_t} u(\tilde{C}_t)$,

$+\beta E_t V(B_t, D_t, Y_{t+1}, x_{t+1})$ subject to the budget constraint

$(1+r_t)D_{t-1} + (1+R_t)W_{t-1} + W_t = \tilde{C}_t + D_t$. This formulation allows for the possibility that any agent holds the government bond at any period. Letting $S_t \equiv T_t - G_t$ denote the primary surplus, the budget constraint in period t is then

$$D_{t+1} = (1+r_{t+1})(D_t - S_t). \tag{14}$$

Market clearings in the good market are rewritten as $C_t + \tilde{C}_t = (1-g)Y_t$, and

$C_{t+1} + \tilde{C}_{t+1} = (1-g)Y_{t+1} = (1-g)(1+x_{t+1})Y_t$.

When private finance is available, agents have no incentive to purchase the government bond in the period when they do not receive income, so that agents purchase the government bond only in the period when they receive income. Equilibrium conditions are finally expressed as three equations:

$$u'(C_t) = \beta E_t\, u'(\tilde{C}_{t+1})\ (1 + R_{t+1})\,, \tag{15}$$

$$u'(C_t) = \beta E_t\, u'(\tilde{C}_{t+1})\ (1 + r_{t+1})\,, \tag{16}$$

and

$$u'(\tilde{C}_t) = \beta E_t\, u'(C_{t+1})\ (1 + R^*_{t+1})\,. \tag{17}$$

Equations (15) and (16) jointly imply that the bank deposit and the government bond are perfect substitutes for agents who are willing to hold these assets, so that without loss of generality we will set $R_t = r_t$. It turns out that the rate of return on the bond is determined to satisfy $1 = E_t\{\beta(1 + x_{t+1})^{-\alpha}\sqrt{1+\kappa}(1 + r_{t+1})\}$. It follows from (15) and (17) that $R_t < R^*_t$ and hence $r_t < R^*_t$, which implies that the rate of return of the government bond r_{t+1} is below the loan interest rate offered by the bank, R^*_{t+1}.

The intermediation cost leads to a reduction of not only the deposit interest rate but also the rate of return on the government bond. Importantly, the lending rate to the government is lower than that to individuals. The model's feature in which only the government is able to lend to others at no transaction cost allows the government to exploit "seigniorage." When the economy is financially "repressed", the rate of return on the government bond tends to be low relative to the growth rate. If so, the government budget is likely to be sustainable even if the primary balance is negative.

We briefly discuss the economy without private finance. If D_t/Y_t grows beyond a certain threshold, the economy reaches a state in which agents hold only the government bond and there is no private lending or borrowing. Agents first purchase the government bond only in the period when they receive income and realize imperfect consumption smoothing, but as the debt-to-GDP ratio reaches a threshold, agents begin to purchase the bond also in the period when they do not receive income, and they realize perfect consumption smoothing. See Bohn (1999, Proposition 2) for details. The rate of return on the bond is, as in the frictionless economy, determined to satisfy $1 = E_t\{\beta(1+x_{t+1})^{-\alpha}(1+r_{t+1})\}$.

If the government adjusts the level of taxation to maintain the average debt-to-income ratio D_t/Y_t below a constant level, its budget will be sustainable, irrespective of the relation between the average growth rate and the average rate of return of the government bond (e.g., Bohn 1998). On the other hand, if the debt-to-income ratio grows beyond a certain threshold, in some finite period the economy reaches a state in which only the government bond is held and there is no private lending and borrowing. In the exchange economy, the endowment grows irrespective of the level of private lending so that fiscal sustainability might coexist with an economy without private lending. However, it is difficult to imagine that the economy without private lending would be successful. Therefore, in the following section we take the debt-to-income ratio, D_t/Y_t, being below a given constant in the long term as a criterion of fiscal sustainability.

We comment on the relation between fiscal sustainability and the no-Ponzi condition when the government implements a policy to stabilize the debt-to-income ratio in every period to satisfy $D_t = dY_t$, with a constant d. Then we derive the discounted value of future outstanding debt as

$$\lim_{N\to\infty} E_t \frac{D_{t+N}}{\prod_{i=1}^{N}(1+r_{t+i})} = dY_t \lim_{N\to\infty} E_t\left[\beta\sqrt{1+\kappa}(1+x_{t+1})^{1-\alpha}\right]^N,$$

which should be zero at the limit. Unlike the standard case of $\kappa = 0$, the condition for the bounded expected utility $E_t\{\beta(1+x_{t+1})^{1-\alpha}\} < 1$ does not guarantee that the no-Ponzi condition will hold. The government may be able to run a Ponzi scheme. If the government were to run a Ponzi game by rolling over debt, it would have to forsake the stable debt-to-income policy, reaching in a finite period a state in which private lending disappeared. Then the no-Ponzi condition would be recovered if the stable debt-to-income policy were adopted.

3. Calibration

3.1 Methodology

To simulate the model, we need dynamic equations for the GDP growth rate, the interest rate on government bond and the debt-to-GDP ratio. We assume that the GDP growth rate follows an AR(1) process:

$$x_{t+1} = \mu(1-\rho) + \rho x_t + \varepsilon_{t+1}, \tag{18}$$

where ε_{t+1} is a random shock following an i.i.d. normal distribution with the standard deviation of σ [9].We have the interest rate on the government bond from Equation (13) with R_t replaced by r_t:

$$r_{t+1} = r + \alpha[\ln(1+x_{t+1}) - \ln(1+\mu)], \tag{19}$$

where $r = R$ is used. We drive the debt-to-GDP ratio from (14) as

$$d_{t+1} = \frac{(1+r_{t+1})}{(1+x_{t+1})}(d_t - s_t), \tag{20}$$

where d_t and s_t are debt and primary surplus divided by GDP_t, respectively, Under some alternative fiscal policy rules for the process of s_t, we simulate the model recursively by combining Equations (18), (19), and (20), starting with d_0. Specifically, we take the following steps.

Step 1. We generate a series of random shocks ε_{t+1} for 100 time periods.

Step 2. We construct x_{t+1} by substituting x_t and ε_{t+1} into Equation (18) recursively with the starting value of $x_0 = \mu$.

Step 3. We construct r_{t+1} by substituting x_{t+1} into Equation (19).

Step 4. We construct d_{t+1} by substituting r_{t+1}, x_{t+1}, and s_t under a particular fiscal policy rule recursively into Equation (20) with a starting value of d_0.

Step 5. We repeat Steps 1–4 10,000 times to obtain the distribution of d_{t+1}, and particularly its mean value.

3.2 Parameters

First, we select the parameters in the utility function. We set the annual discount factor β at 1/1.02=0.9803, a figure often applied to the Japanese economy[10]. The inverse of the elasticity of intertemporal substitution, α, plays a key role in the model, relating the interest rate to the GDP growth rate. To set α, we regressed the nominal government bond yield $\hat{r}_r$ on the nominal GDP growth rate $\hat{x}_t$ using OLS. We chose a sample period from 1981 to 2005 to exclude the period with a regulated interest rate. The estimation result is as follows:

$$\underset{(0.004)\ (0.089)}{\hat{r}_t = 0.020 + 0.668\hat{x}_t}, \quad Adj.R^2 = 0.696, \tag{21}$$

where the numbers in the parentheses are standard errors. $Adj.R^2$ denotes the coefficient of determination adjusted for degrees of freedom. Based on Equation (21), we set α at 0.668, or equivalently the elasticity of intertemporal substitution at 1.497. The goodness of fit is reasonably high. The 95 percent confidence interval for the coefficient on $\hat{x}_t$ implies that the elasticity of substitution lies between 1.17 and 2.07, which is consistent with much of the recent micro and macro evidence for the Japanese economy, reporting the elasticity in the range between one and two[11].

Next, we choose the parameters of technology. We set the average real GDP growth rate μ to 0.025 that is the average growth rate over the period of fiscal years 1981 to 2004. The mid-term economic outlook published by the Japanese government predicts that the growth rate will gradually accelerate from fiscal year 2006, reaching 2.5 percent in fiscal year 2011 (Cabinet Office, 2007)[12]. The parameters in the autoregressive process of the real GDP mainly affect the probability that the debt-to-GDP ratio decreases from the initial level. We estimated the AR(1) process, applying OLS to the annual real GDP growth rate data over the period of fiscal years 1981 to 2004:

$$\underset{}{x_t} = \underset{(0.006)}{0.014} + \underset{(0.195)}{0.451}x_{t-1}, \quad Adj.R^2 = 0.166, \tag{22}$$

where x_t is the real GDP growth rate. The estimated value of ρ is positive and significant, consistent with the model's assumption that the growth rate of income follows a Markov process. We set ρ to 0.451 and σ to 0.01917, where the latter is the root MSE of the above regression.

Third, we set the financial intermediation cost, κ, to be the interest margin between bank loans and deposits. We calculate the average interest margins of domestic banks by taking differences between the average loan interest rate (loan interest income divided by loans outstanding) and the average deposit interest rate (deposit interest expenses divided by deposits outstanding) reported in their financial statements[13]. We obtain 1.88 percent for 1975–79, 2.28 percent for 1980–89, 2.08 percent for 1990–99, and 2.15 percent for 2000–06. The average interest margins for

1980–2006, when financial liberalization was promoted, is 2.17 percent. We set κ at 0.022. Although the interest margin may reflect a default risk, we interpret the interest margin as the intermediation cost, because the intermediation cost should include the costs of defaults[14].

3.3 Fiscal policy rules

Bohn (1998) proposes a fiscal policy under which the government increases a primary surplus when the debt-to-GDP ratio increases, and vice versa. While he claims that the US experience was consistent with this rule, the Japanese government does not seem to have followed it, at least during the past 30 years[15]. Rather, as is shown by Figure 2, we observe a positive correlation between the primary balance and the GDP growth rate. This observation will motivate us to guess that the Japanese primary balance is an increasing function of the GDP growth rate[16]. Regressing the primary balance as a proportion of nominal GDP to the growth rate of *nominal* GDP over the period of fiscal years 1981–2004, we obtained the following result:

$$\underset{}{s_t} = \underset{(0.006)}{-0.053} + \underset{(0.131)}{0.997\hat{x}_t}, \quad Adj.R^2 = 0.713. \tag{23}$$

Given the high goodness of fit, we can safely say that the primary balance actually depends heavily on the nominal GDP growth rate.

We used the nominal GDP growth rate and not the real one because the government is likely to determine expenditures considering tax revenues that are strongly affected by nominal GDP. In fact, the nominal GDP growth rate provides a better fit than the real GDP growth rate[17]. This policy rule is not necessarily inconsistent with our theoretical model defined in real terms. Based on Equation (23), we assume the following fiscal policy rule:

$$s_t = -\eta + 1.0 x_t, \tag{24}$$

where the intercept η is a free parameter that is intended to capture the government's fiscal stance. Equation (24) implies that the primary balance as a proportion of GDP increases by 1 percent if the GDP growth rate increases by 1 percent. We set η to 0.025 so that the primary balance becomes zero if the growth rate is 2.5 percent, considering the government's goal for a fiscal surplus in fiscal year 2011. The figure of 0.025 is higher than the constant term of Equation (23), –0.053, by 2.8 percent, reflecting the government's new policy rule, which will differ from its past behavior. It is the advantage of calibration that we can incorporate a change in rules into calculations, as well as utilize past variables. By increasing η we also investigate whether the sustainability condition is met for each value of η.

The initial value of the debt-to-GDP ratio is set at 1.72, the actual gross debt (net of government short-term securities) of the general government as a proportion of GDP at the end of 2005[18].

As a criterion for fiscal sustainability, we set the threshold value of the debt-to-GDP ratio at this initial value. It is reasonable to judge fiscal sustainability based on whether the future expected value of the debt-to-GDP ratio does not exceed its initial value. Our criterion of fiscal sustainability coincides with the definition of Blanchard et al. (1990), which is that the ratio of debt-to-GDP eventually converges back to its initial level over the finite horizon. However, the value of 1.72 is quite high, from the perspectives of both Japanese history and cross-country comparisons. It should be noted that if economic conditions change, such as the savings rate and/or the long-run growth rate, the target level of 1.72 may become too high to be a critical value for sustainability[19].

Finally, we comment on the definition of government debt. Some argue that instead of gross debt, net debt of government financial assets should be used. Doi (2006), however, points out that financial assets held by social security funds should be subtracted from government debt only if we forecast government expenditures taking into consideration future social security outlays. Because we do not forecast future social security outlays, we do not subtract financial assets of social security funds from the government bond. Doi (2006) also points out that the decision of whether financial assets held by the central or local governments should be subtracted from government assets depends on political judgments of whether those assets will be used to repay debt. We understand that government financial assets are not used to repay debt.

Table 1 summarizes the parameters that we use for the baseline calibration.

3.4 Baseline results

Column 1 in Table 2 shows the baseline result under the assumption that the primary balance averages zero. Hereafter we evaluate all variables in real term. The mean interest rate on government debt is 2.57 percent, slightly higher than the average GDP growth rate. Figure 3.A shows that the average debt-to-GDP ratio gradually increases from the initial level of 1.72, reaching 1.896 in 100 years. The probability that the debt-to-GDP ratio reaches a level below the initial value in 100 years is 39.7 percent. The government debt is not sustainable based on our criterion.

We investigate how much primary surplus is necessary to make government debt sustainable. We find that a primary surplus of 0.1 percent of GDP raises the average debt-to-GDP ratio to 1.791 in 100 years, but a primary surplus of 0.2 percent of GDP decreases the average debt-to-GDP ratio, reaching 1.681 in 100 years[20]. In the latter case, the expected debt-to-GDP ratio decreases, although the probability that the debt-to-GDP ratio increases from the initial level is still as high as 44.4 percent. A primary surplus of 0.2 percent of GDP makes the government debt sustainable.

We calculate the primary surplus required for decreasing the debt-to-GDP ratio with a probability of 95 percent. In Figure 3.B, increasing the average primary surplus further, we find that if the targeted primary surplus is 1 percent of GDP, the debt-to-GDP ratio decreases from the initial level with a probability of 94.4 percent. If the targeted primary surplus is 1.1 percent of GDP, the debt-to-GDP ratio decreases with a probability of 96.3 percent. If the government is to achieve a primary surplus of 1 percent of GDP by raising consumption taxes, it must raise the tax rate by about 2.5 percentage points given that its expenditure as a proportion of GDP is constant[21].

3.5 Results under alternative parameters

In this section, we make several alternative assumptions to examine changes in the baseline results. Table 3 shows the results.

In Case 1, we set the financial intermediation cost, κ , to zero. The average interest rate of the government bond is 3.7 percent, 1.2 percentage points higher than the GDP growth rate. The real interest rate is so high that the average debt-to-GDP ratio reaches 5.599. With an almost 100 percent probability, the debt-to-GDP ratio increases from the initial level. To stabilize the debt-to-GDP ratio below the initial level, the primary surplus as a proportion of GDP must be 2.1 percent. How can we interpret this result? The average nominal interest rate of long-term government bonds is 5.0 percent over the period 1981–1998, the period prior to the zero-interest-rate policy, while the average nominal GDP growth rate over the same period is 4.2 percent, 0.8 percentage points lower than the interest rate (see Figure 1). However, considering the fact that the average funding cost of the government is no more than the long-term interest rate, it does not seem likely that the interest rate of government bond is 1.2 percentage points higher than the GDP growth rate.

In Case 2, we set the inverse of the elasticity of intertemporal substitution, α , at 1, which is often assumed in the literature of dynamic stochastic general equilibrium analysis. The average interest rate of government bond is 3.42 percent, higher than the baseline result of 2.57 percent. The debt-to-GDP ratio increases rapidly and reaches 4.204 in 100 years. The government debt is not sustainable. This case does not seem likely either, considering the past relationship between the interest and growth rates.

We have so far assumed that the Japanese economy achieves a GDP growth rate of 2.5 percent. Anticipating possible declines in labor force and technological progress due to the future population aging, the growth rate may be lower than 2.5 percent. To consider this possibility, we set the GDP growth rates at 2 percent, 1 percent, and 0 percent in that order. We also assume that the government achieves a zero primary balance under each growth rate. The results are shown in Cases 3A, 3B and 3C, respectively. The average interest rates of government bonds decrease to 2.24 percent, 1.57 percent, and 0.9 percent as the growth rates decrease to 2 percent, 1 percent, and 0 percent, respectively. The gaps between the interest rate and the growth rate widen to 0.24 percent, 0.57 percent, and 0.9 percent, while it is 0.07 percent in the benchmark case. The difference in the results comes from the fact that we set α to 0.668, put differently, the elasticity of intertemporal substitution to be larger than unity. The debt-to-GDP ratios increase from the initial level, reaching 2.206, 3.081 and 4.288 in 100 years, which suggests that the government debt is not sustainable in any case. The primary surpluses necessary to reduce the expected debt-to-GDP ratio from the initial value are 0.5 percent, 1 percent and 1.6 percent of GDP, respectively. The primary surpluses necessary to reduce the debt-to-GDP ratio from the initial value with a probability of 95 percent are 1.3 percent, 1.9 percent and 2.5 percent of GDP, respectively. As the GDP growth rate declines, the gap between the interest and growth rates widens and the primary surplus necessary for the sustainability condition increases. Ihori et al. (2007) make a more pessimistic simulation under the assumptions that the rate of technological progress is 0 percent and that the labor force declines, as we explain in detail in the next section.

Finally, we consider the possibility that the fiscal policy is featured by a primary surplus that is a nonlinear function of the GDP growth rate. When the GDP growth rate is low, primary balances may worsen extremely because of expansive fiscal policy measures. It may be difficult to restore primary balances by increasing taxes or decreasing government expenditure, which would cause significant redistribution. Progressive income tax may also cause a nonlinear relationship between primary surplus and GDP growth rate. Regressing the primary surplus as a proportion of nominal GDP on the nominal GDP growth rate and its squared values, we obtained the following concave equation:

$$\underset{}{s_t} = \underset{(0.006)}{-0.054} + \underset{(0.343)}{1.479\hat{x}_t} - \underset{(4.846)}{7.321\hat{x}_t^2}, \quad Adj.R^2 = 0.729. \tag{25}$$

We employ the above fiscal policy rule by adjusting the constant term so that the primary balance becomes zero under a 2.5 percent real GDP growth rate.

Case 4 shows that the probability that the debt-to-GDP ratio decreases from the initial level in 100 years is 22.2 percent, lower than the baseline result (39.7 percent). This result is obtained from our assumption that primary balance deteriorates more than proportionately when the real GDP growth rate declines.

3.6 Comparisons with previous studies

In this subsection, we compare our calibration results with previous studies on the sustainability of Japanese government debt. Ihori et al. (2007), directly comparable to ours, calculate the interest rate from the growth model of overlapping generations. They project a primary balance that is necessary to maintain the debt-to-GDP ratio. Their estimated primary balance (3.9 percent of GDP on average) is greater than ours primarily because the gap between the interest and growth rates exceeds almost 3 percentage points, which is much larger than our result. The main reasons for this difference are as follows. First, they do not consider financial intermediation costs. Second, they set the inverse of the elasticity of intertemporal substitution at a higher value than ours. Third, they assume a lower GDP growth rate than ours[22].

Dekle (2005) examines the small-open version of the growth model and finds that the government debt is not sustainable unless the primary surplus increases substantially to almost 5 percent of GDP over the next decade or so. His projection assumes a wide gap between the interest rate (6 percent) and the TFP growth rate (1.2 percent).

Broda and Weinstein (2005) and Doi (2006) make simulations using exogenous pairs of economic growth rates and interest rates. Broda and Weinstein (2005) project that the government revenues as a proportion to GDP must increase moderately[23]. Doi (2006) points out that more government revenues are necessary than Broda and Weinstein's results if financial assets held by the central and local governments are not subtracted from their debt and if we consider the recent financial deterioration. Note that the simulation results of Broda and Weinstein (2005) and Doi (2006) are difficult to compare with our results directly because both of them ignore general equilibrium consideration between growth and interest rates.

Oguro (2006) applies the approach of "deficit gamble" (Ball et al., 1998) to the Japanese economy, finding that if the primary deficit is zero, the probability that the debt-to-GDP ratio will fall below its initial value in 100 years is 49.4 percent, higher than our baseline result (39.7%). His methodology is to estimate the joint distribution of the interest rate and the GDP growth rate over the period 1966–2005 and to project the estimated debt-to-GDP ratio to the future. Notably, the sample period includes the high-growth era when the interest rate is lower than the average growth rate. His optimistic prediction appears to come from the chosen sample period.

4. Conclusions

To answer the question of whether government debt is sustainable in Japan, we have calibrated a dynamic stochastic general equilibrium (DSGE) model. By incorporating financial intermediation costs into the theoretical model, we have succeeded in explaining the actual relationship between the interest rate and the GDP growth rate observed for the Japanese economy in some consistent manner. Our approach has also made it possible to assume some combinations of the interest rate and the economic growth rate on theoretical grounds. Our approach will remain an effective analytical tool even when the future potential growth rate changes. Our calibration has also enabled us to show the variability of our forecasts.

In investigating the fiscal sustainability by calibrating a DSGE model, it is extremely important to specify the relationship between the interest rate on government bond and the GDP growth rate in a manner that is consistent with the actual data. With parameter sets consistent with the Japanese economy, it makes little difference whether government bonds are treated as safe or risky assets, while it is essential to introduce financial intermediation costs to calibrate the model with realistic interest rates.

Our results can be summarized as follows. When the growth rate is 2.5 percent, the average interest rate is 2.57 percent under the assumption that financial intermediation costs remain at the same level as in the past. If the targeted primary surplus averages zero, the debt-to-GDP ratio gradually increases from the initial level of 1.72 to 1.896 in 100 years, suggesting that the government debt is not sustainable. To make it sustainable, the targeted primary surplus must be 0.2 percent of GDP. Lowering the GDP growth rate to 2, 1, and 0 percent and assuming that the government runs a zero primary surplus for each growth rate, we find that the average interest rate for government bond decreases but the gap between the interest rate and the growth rate widens, so that the government faces greater difficulty in meeting the sustainability condition.

It should be noted that our calibration results depend on several assumptions. First, we assume that government outlays as a proportion to GDP are constant. If we consider an increase in social security expenses associated with population aging, we may say that government revenues necessary to meet the sustainability condition will exceed our estimate. Next, we have argued the fiscal sustainability issues under the criterion that the debt-to-GDP ratio decreases from its 2005 level, 1.72. If we consider a decrease in savings rate, which will be inevitable with population aging, we may have to set a more stringent criterion than 1.72. Furthermore, we do not consider the possibility that the interest rate on government bonds incorporates the risk of an increasing debt-to-GDP ratio. If we consider this possibility, the primary balance necessary to meet the sustainability condition may be higher than our estimate. The effect of the risk of fiscal insolvency on the interest rate depends on how we model the economy after the fiscal insolvency, which is an important direction for future research. Finally, it should be noted that our model is an exchange economy with growth rates being exogenous. Generally, the accumulation of government debt may deter private capital accumulation and have a negative impact on economic growth.

We must not hastily conclude that a reduction of financial intermediation costs due to financial innovation will raise the interest rate and tighten the government budget. We have employed an exchange economy model without production to present a simple model that consistently explains the actual relationship between the interest rate and the GDP growth rate. The next task is to extend the model to an endogenous growth model with production and to calibrate it to the Japanese economy. In the endogenous growth model, a reduction of financial intermediation costs may possibly raise the economic growth rate, resulting in an increase in tax revenues and a primary surplus.

Our model is a standard asset-pricing model incorporating financial intermediation costs. To incorporate various other extensions that have been tried to explain the safe asset return, such as the decoupling of the degree of relative risk aversion and the elasticity of intertemporal substitution (Weil, 1989, among others), is left for future studies.

Acknowledgement

We thank two anonymous referees, Yasushi Iwamoto, Toshihiro Ihori, Takatoshi Ito, Akira Okamoto, Masao Ogaki, Etsuro Shioji, Makoto Saito, Toru Nakazato, Tomoaki Yamada, Tsutomu Watanabe and seminar participants at the 9th Macroeconomics Conference, Hitotsubashi University, Fiscal Studies Group at the Institute of Statistical Research for their valuable comments. Sakuragawa thanks Kikawada Foundation for financial support. Remaining errors are ours.

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Table 1. Parameters

Preference		Technology			Financial Intermediation	Fiscal Policy Rule		Initial Condition
Discount Rate	Inverse of Elasticity of Intertemporal Substitution	Mean Growth Rate	Serial Correlation	Standard Deviation	Intermediation Cost	Coefficient on Growth Rate	Constant Term	Debt / GDP
β	α	μ	ρ	σ	κ	γ	η	d_0
1/1.02	0.668	0.025	0.451	0.01917	0.022	1	0.025	1.72

Table 2. Calibration Results

Average primary surplus-to-GDP Ratio (%)	0	0.1	0.2	0.5	1	1.1
Average real interest rate (%)	2.57	2.57	2.57	2.57	2.57	2.57
Average debt-to-GDP ratio in 100 years	1.896	1.791	1.681	1.362	0.852	0.737
Probability that debt-to-GDP ratio in 100 years is less than its initial value (%)	39.7	47.0	55.6	76.0	94.4	96.3

Note: We adjust η to realize each average primary surplus-to-GDP ratio.

Table 3. Calibration Results under Alternative Assumptions

Case		1	2	3A	3B	3C	4
	Benchmark	Zero Financial Intermediation Cost	Unitary Elasticity of Intertemporal Substitution	2% GDP Growth Rate	1% GDP Grwoth Rate	0% GDP Grwoth Rate	Primary Surplus as a Quadratic Function of GDP Grwoth R
Parameters Changed		$\kappa=0$	$\alpha=1$	$\mu=0.02$	$\mu=0.01$	$\mu=0.0$	
A. In case average primary surplus is zero							
Average real interest rate (%)	2.57	3.70	3.42	2.24	1.57	0.90	2.57
Average debt-to-GDP ratio in 100 years	1.896	5.599	4.204	2.206	3.081	4.288	2.226
Probability that debt-to-GDP ratio is less than its initial value (%)	39.7	0.0	0.0	22.6	3.9	0.3	22.2
B. Primary surplus-to-GDP ratio necessary to make average debt-to-GDP ratio in 100 years less than its initial value (%)	0.2	2.1	1.6	0.5	1.0	1.6	
C. Primary surplus-to-GDP ratio necessary to make the probability that debt-to-GDP ratio in 100 years less than its initial value 95% or more (%)	1.1	3.0	2.1	1.3	1.9	2.5	

Note: We adjust η to realize each average primary surplus-to-GDP ratio.
For the fiscal policy rule of case (4), see the text.

Figure 1 Yields to Government Bond and Nominal GDP Growth Rate

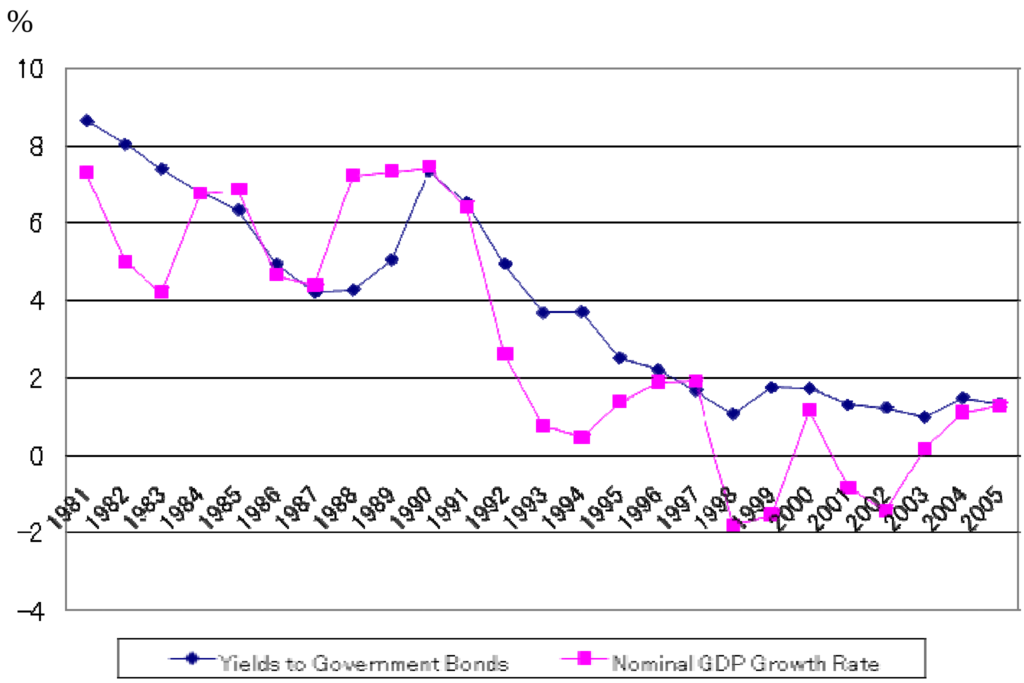


Figure 2 Primary Balance as a Proportion to GDP and Nominal GDP Growth Rate

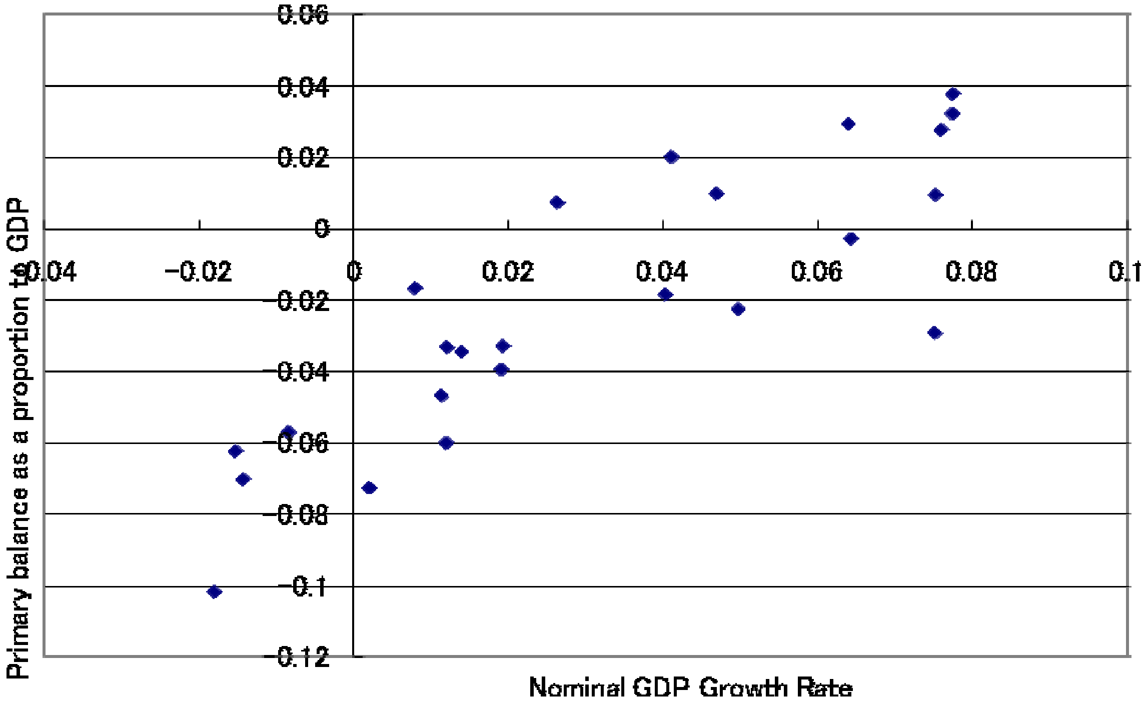
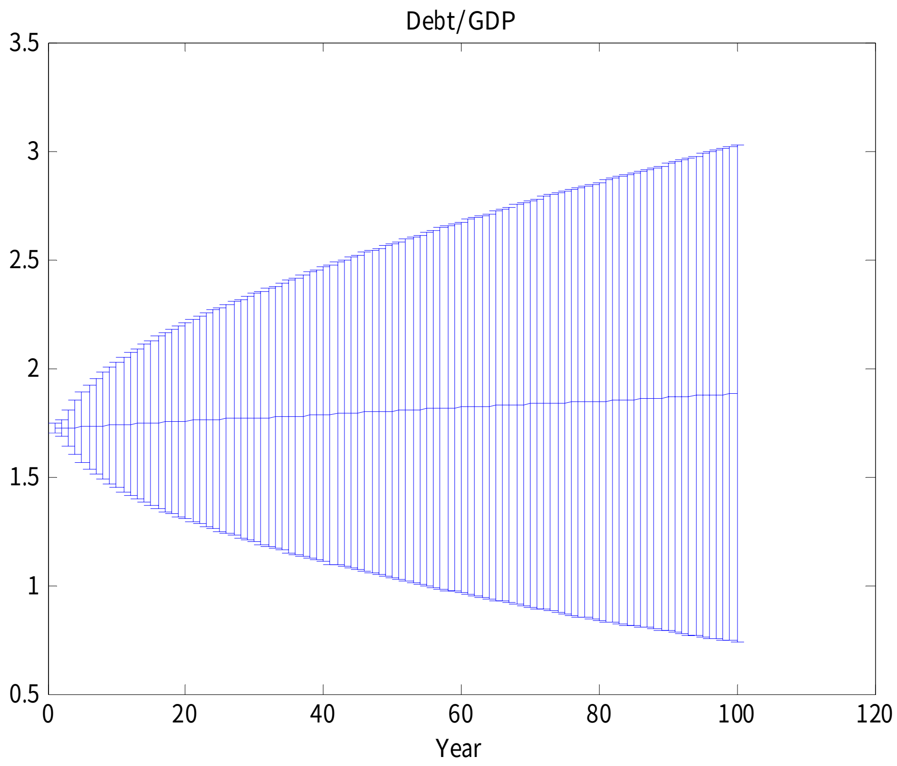


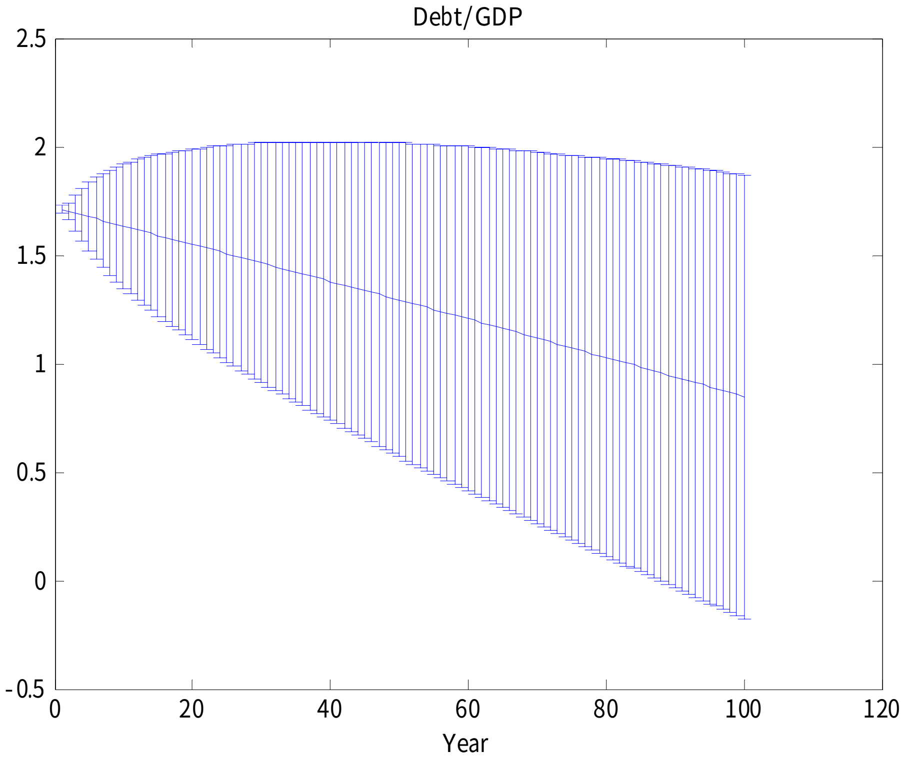
Figure 3 Calibration Results (Government Debt-to-GDP ratio)

A. (Benchmark Case)：Primary balance is zero on average.



Real line is the average. The vertical lines show the range within two times standard errors.

B. Primary surplus is 1.0% of GDP on average.



Real line is the average. The vertical lines show the range within two times standard errors.

Footnotes

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[1] Taking the averages over the whole sample period of 1981–2005, we find that the gap between the nominal interest rate on the government bond and the nominal GDP growth rate is 1.0 percentage point. However, considering the possibility that the nominal interest rate remains at a higher level than the equilibrium rate under the zero-interest-rate policy, we believe that a gap of 1.0 percentage point is not appropriate as a benchmark argument for fiscal sustainability.

[2] At the beginning of 2005, politicians and government officials had a heated debate on "growth rate versus interest rates." When investigating fiscal sustainability, it is of extreme importance to understand the relationship between the interest rate on government bonds and GDP growth rate. The government released reports for fiscal sustainability for possible several pairs of economic growth rates and interest rates, and provided a consensus that if the pair of high economic growth rates and low interest rates is realized, the likelihood of fiscal sustainability increases. The government, however, did not explain which pair is most consistent with the potential of the Japanese economy.

[3] Bohn (1999) demonstrates that debt-financing strategies that exploit low interest rates lead to different welfare implications between the two approaches. Blanchard and Weil (2001) investigate various models of the two approaches, and show that the feasibility of Ponzi games varies with models, not necessarily depending on the average rate of return on bonds versus the growth rate.

[4] Ihori et al. (2002) also apply the approach of Hamilton and Flavin and report that the sustainability condition is not satisfied for the period of 1957–99, when the gap between the interest rate and the GDP growth rate was 4 percent or more.

[5] Broda and Weinstein (2005) report an optimistic projection for the sustainability, whereas Doi (2006) provides a pessimistic one. The difference in their results lies in the difference in whether government assets are included or not.

[6] Aiyagari and Gertler (1991) and Heaton and Lucas (1996) construct models with intermediation costs and heterogeneous agents to resolve the Mehra–Prescott puzzle.

[7] Note that it is easily possible to verify that rational agents never borrow in the period when they receive income and never lend in the period when they do not that we skip this part in the formulation.

[8] Rigorously, we may formulate this as

$$V(B_{t-1}, Y_t, x_t) = \max_{C_t, W_t} u(C_t) + \beta \int_{\underline{x}}^{\bar{x}} \tilde{V}(W_t, Y_{t+1}, x_{t+1}) g(x_{t+1} | x_t) dx_{t+1} .$$

[9] The assumption of normal distribution may raise concern about the possibility of negative gross growth rates. In practice, however, this is not the case. We simulated the model 10,000 times over 100 periods and found that the minimum value of the gross growth rate was 0.9143, far above zero, given the values of μ and σ assumed as below.

[10] For example, Sugo and Ueda (2007) set the quarterly discount factor at 0.995. Fujiwara et al. (2005) set the annual discount factor at 0.99.

[11] Though some of the studies in the 1990s report the elasticity of intertemporal substitution below 1 (e.g., Kaneko, 1991; Akagi, 1995; and Uemura, 1997), more recent studies report it as being between 1 and 2. Abe and Yamada (2005) analyze household micro data, obtaining the estimated elasticity of 1.809. Hatano and Yamada (2006), using macro data, obtained the estimated elasticity above 1 in most of the specifications (see also Hamori, 1996; Okubo, 2003; Fuse, 2004; Iiboshi et al., 2006; and Sugo and Ueda, 2007).

[12] See the "transition-to-the-new-growing-economy scenario" in Cabinet Office (2007).

[13] Domestic banks include city banks, long-term credit banks, trust banks, first-tier and second-tier regional banks.

[14] Though we do not specify the intermediation cost, we can regard it as, for example, the expected judicial cost for forfeiting borrowers' assets in case of defaults, which depends on the probability of default. The costly-state-verification model posits that the loan interest rate is equal to the deposit interest rate plus the verification cost multiplied by the default probability (e.g., Gale and Hellwig, 1985, and Williamson, 1986).

[15] Doi and Ihori (2002), using Japan's data over the period of 1965–2000, provide evidence that Bohn's rule did not hold in Japan. Ito et al. (2006) also show that Bohn's rule did not hold after 1975.

[16] One may think that we should use the GDP gap, i.e., the difference between potential and actual GDP, instead of the GDP growth rate to capture the role of a fiscal policy as a built-in stabilizer. However, we choose the growth rate because most government expenditure, including social security expenditure, does not depend on the growth rate, while tax revenues depend on it strongly. If we used the GDP gap, we would implicitly assume that the government will increase taxes to finance primary deficits caused by the decline in the potential growth rate (e.g., an increase in social security expenditures arising with aging), which would be politically difficult and unrealistic due to its significant redistribution effect among various interest groups (Alesina and Drazen, 1991).

[17] Regressing the primary surplus as a proportion of GDP on the real GDP growth rate, we get

$$\underset{}{s_t} = \underset{(0.012)}{-0.039} + \underset{(0.362)}{0.710}x_t, \quad Adj.R^2 = 0.110$$

Following this estimation result, we assume that the fiscal policy follows the rule $s_t = 0.7x_t - 0.0175$, which implies that the primary surplus is zero in case of the 2.5 percent growth rate. The calibration result under this policy rule was similar to that in Table 2, Column 1 (The expected government debt-to-GDP ratio is 1.893 in 100 years, and the probability that the debt-to-GDP ratio in 100 years is below its initial value is 37.0 percent).

[18] Total debt (excluding short-term government bonds) of the general government at the end of 2005 is 861705.7 billion yen (Flow-of-fund statistics by Bank of Japan).

[19] Ball et al. (1998) analyze the U.S. fiscal sustainability using 1.0 or 1.5 as the critical value of the government debt as a proportion of GDP.

[20] In the following calibrations, we change η by 0.001.

[21] The consumption tax rate was 5 percent and consumption tax revenues were 2.0 percent of nominal GDP in fiscal year 2006.

[22] They set the inverse of the elasticity of intertemporal substitution to 2.5. In their baseline scenario, the annual rate of TFP growth is zero from 2003 on and the labor force declines.

[23] In their "Case 2," for example, the government revenues as a proportion to GDP must increase by 2.7 percentage points. In "Case 2", the forecast for the population is based on Faruqee and Mühleisen (2001), the GDP growth rate is assumed to be 2 percent, the interest rate assumed to be 4 percent, and both the per-capita government transfers to older generations and the per-capita government expenditures to young generations increase at the same rate as GDP.