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National Currency-Based International Input-Output Analysis: Data Construction and Model Structure

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Abstract

This paper aims at constructing national currency-based international input-output tables in constant prices and developing the theoretical framework of a national currency-based international input-output model. Similar to most multi-sectoral models, the model has micro foundations. Departing from perfect competition, however, international oligopolistic competition in price is incorporated. Since the model is a global model based on national currencies, we can apply it not only to analyze global economic issues such as global warming and international trade but also to evaluate national economic policies such as monetary and fiscal policies within a unified international framework. In these respects, the model is quite unique and can provide a new approach to empirical economic analysis.

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1. Introduction

This paper constructs national currency-based international input-output tables in constant prices and presents the theoretical structure of a national currency-based international input-output model.

Since World War II, economic interdependence of nations has been strengthened through trade and investment. Project LINK is a pioneering global macroeconometric model which quantifies the effects of economic policies and/or changes in exogenous economic environment on the world economy.[1] Subsequently, many institutions and scholars construct multi-county macroeconometric models such as the International Monetary Fund's Global Economy Model (Pesenti, 2008), Fair's (1994) Multi-Country Model, Taylor's (1993) Multi-Country Model, and so on. However, trade is transactions of goods and the degree of globalization differs sector by sector. Therefore, global macroeconometric models are not necessarily adequate. Instead, global models at sector level is more appropriate for analyzing the current world economy. Regarding multi-country multi-sectoral models, the following four types of models have been developed: 1) computable general equilibrium (CGE) model such as the Michigan model (Deardorff and Stern, 1986), the GTAP model (Hertel, 1996) and the G-Cubed model (McKibbin and Wilcoxen, 1999)[2], 2) the INFORUM system which interlinks national input-output models with a trade linkage model (Almon, 1991; Uno, 2002), 3) single-period international input-output

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[1] Project LINK, initiated in 1968 by Professor Lawrence Klein, is currently maintained at the United Nations.

[2] Among these example models, parameters of the G-Cubed model are econometrically estimated.

model (Torii et al. 1989; Kosaka, 1994; Yano and Kosaka, 2003), and 4) price-linked international input-output model (Yano and Kosaka, 2008). However, the first three models have shortcomings: a typical CGE model lacks statistical foundations of parameters; the INFORUM system might have inconsistency between classifications in input-output tables and trade matrix; a single-period international input-output model has limitations in specifications and estimation of behavioral equations due to the use of only a single-period international input-output table. A price-linked international input-output model improves the flaws of these three model, yet it has a drawback: that is, a currency problem. The currency problem is inconsistency among national currencies and a unified currency applied in international input-output tables. Economic agents (including policy makers) make their decisions by focusing on economic performance of their economies in their currencies rather than a foreign currency. However, international input-output tables are denominated in a specific currency. This shows that we must build a national currency-based model in order to analyze economic issues. To do this, this paper shows an approach to compile international input-output tables in constant prices and national currencies. In addition, the structure of a national currency-based international input-output model is also presented. Holding statistical foundations of parameters and consistency between classifications of industry and trade, the national currency-based international input-output model enables us to analyze global and sectoral effects effects of each economy's policies.

The rest of this paper consists of three sections. Section 2 illustrates the method to construct national currency-based international input-output tables in constant prices. Section 3 presents the model structure. Finally, section 4 provides concluding remarks.

2. Construction of National Currency-Based International Input-Output Tables in Constant Prices

2.1 The Structure of an International Input-Output Table

A national currency-based international input-output model is built on national currency-based international input-output tables in constant prices. Since most international input-output tables are in current prices and denominated in a specific currency, it is imperative to transform them into the tables in constant prices and denominated in national currencies. Although our approach to construct the tables of interest can be applied to any international input-output tables which are complied for at least two years, we explain the procedure by using the Asian International Input-Output Table as a benchmark.

The Asian International Input-Output Table covers the ten economies (Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, Taiwan, South Korea, Japan, the United States) and is available for the years 1985, 1990, 1995, and 2000 (Institute of Developing Economies 1993, 1998, 2001; Institute of Developing Economies-Japan External Trade Organization 2006a, 2006b). Figure 1 shows the structure of the 2000 Asian International Input-Output Table.[3] Its fundamental

[3] The structure of the tables for the other years is slightly different from that for the year 2000. For details, see Institute of Developing Economies (1993, 1998, 2001) and Institute of Developing

structure is the same as that of a single country input-output table: however, exports to and imports from the third world as well as trade related variables (i.e., international freight and insurance plus import duties) are added. It is worth noting that final demand (*F*) is further disaggregated into the following sub-categories: private consumption (*CP*), government consumption (*CG*), investment (*IN*), and inventories (*IV*). For later purpose, an international input-output table is split into the following four parts as in Figure 2: Part A for intermediate goods, Part B for final demand, exports to the third world, and statistical discrepancies, Part C for output, and Part D for value added.

Figures 1 and 2

2.2 Currency Conversion

International input-output tables are typically denominated in a single currency: e.g., the Asian International Input-Output Tables for the four years are evaluated in U.S. dollars. In contrast, national currency-based international input-output tables consist of variables in currencies *h* (economy which supplies goods) and *k* (economy which consumes goods). Following the double deflation technique, intermediate goods (Part A of Figure 2), final demand, exports to the third world, statistical discrepancies (Part B of Figure 2), and output (Part C of Figure 2) are denominated in currency *h*. On the contrary, value added (Part D of Figure 2) is converted into

Economies-Japan External Trade Organization (2006a, 2006b).

that in currency *k*. In order to hold the consistency between the summation of inputs and demands, intermediate goods (Part A of Figure 2) are evaluated by currency *k* as well: i.e., we have two sets of intermediates (one is evaluated by currency *h* and the other is by currency *k*). Consequently, the following five parts should be obtained: 1) intermediates evaluated by currency *h*, 2) intermediates evaluated by currency *k*, 3) final demand, exports to the third world, and statistical discrepancies evaluated by currency *h*, 4) output evaluated by currency *h*, and 5) value added evaluated by currency *h*.

2.3 Deflation

In order to deflate an input-output table, the double deflation technique is normally applied. By contrast, Hoen (2002) develops a different deflating procedure which uses the RAS method. As Hoen (2002) points out, his approach would be more proper than double deflation. However, the RAS approach requires various data in constant prices in advance of deflation. According to Hoen (2002, p.78), the following data in constant prices are required for deflating international input-output tables: sectoral output, sectoral exports to and imports from the third world, sectoral value added, and totals of final demand components of each economy which consists the corresponding tables. On many occasions, it is not easy to obtain the required data even for developed countries. Therefore, we employ Yano and Kosaka's (2008) simpler approach which uses the principles of double deflation. The double deflation method requires price data for each sector and economy prior to deflation: however, it is rare to find proper set of these data. Viewing sectoral GDP deflator as the corresponding sector's value added deflator in the international input-output framework, Yano and Kosaka (2008) obtain sectoral price equations of all economies by backtracking the double deflation method and compute the values by solving the system of the resultant price equations.

2.4 The Detailed Procedure

The procedure of constructing national currency-based international input-output tables in constant prices is described as follows:

Step 1: Unification of sector classification

Sector classifications of international input-output tables and GDP deflators are not always identical. Therefore, we unify the sector classifications of these data, if necessary.

Step 2: *Construction of international input-output tables in current prices and national currencies*

Prior to deflating international input-output tables, we construct those in current prices and national currencies. Expressions of currency conversions are presented in Table 1. It is worth noting that intermediate goods in currency k are computed by converting intermediate goods in currency h into those in currency k since international input-output tables are deflated by currency h.

Table 1

Step 3: *Computation of sectoral prices by using the corresponding sector's GDP deflators*

Following double deflation, value added deflator is written as:

$$PVA_j^k = \frac{XX_j^k - \sum_h \sum_i XK_{ij}^{hk} - GA_j^k - OA_j^k - WA_j^k}{\frac{XX_j^k}{P_j^k} - \sum_h \sum_i \frac{XH_{ij}^{hk}}{P_i^h} \frac{e^{k*}}{e^{h*}} - \frac{GA_j^k}{PIM^k} - \frac{OA_j^k}{PIM^k} - \frac{WA_j^k}{PIM^k}} \tag{1}$$

where PVA_j^k is value added deflator in sector *j* of economy *k*, XX_j^k is output in sector *j* of economy *k* in current prices and currency *k*, XK_{ij}^{hk} is good *i* in sector *j* of economy *k* delivered from economy *h* in current prices and currency *k*, GA_j^k is exports to the European Union in sector *j* of economy *k* in current prices and currency *k*, OA_j^k is exports to Hong Kong in sector *j* of economy *k* in current prices and currency *k*, WA_j^k is exports to the rest of the world in sector *j* of economy *k* in current prices and currency *k*, P_j^k is price in sector *j* of economy *k*, XH_{ij}^{hk} is good *i* in sector *j* of economy *k* delivered from economy *h* in current prices and currency *h*, P_i^h is price in sector *i* of economy *h*, e^{k*} is the base-year exchange rate of economy *k*, e^{h*} is the base-year exchange rate of economy *h*, and PIM^k is import deflator of economy *k*. Rearranging equation (1) yields equation for P_j^k as:

$$P_j^k = \frac{XX_j^k}{B_j^k} \tag{2}$$

where

$$B_j^k = \sum_h \sum_i \frac{XH_{ij}^{hk}}{P_i^h} \frac{e^{k*}}{e^{h*}} + \frac{GA_j^k}{PIM^k} + \frac{OA_j^k}{PIM^k} + \frac{WA_j^k}{PIM^k} + \frac{XX_j^k - \sum_h \sum_i XK_{ij}^{hk} - GA_j^k - OA_j^k - WA_j^k}{PVA_j^k} \tag{3}$$

Stacking equation (2) of all sectors and economies and solving the resultant simultaneous system give values for sectoral prices of all economies in national currencies.

Step 4: *Deflation of international input-output tables in current prices and national currencies*

Applying the double deflation technique, we deflate intermediate goods, final demand components, exports to the third world, statistical discrepancies, and output at the sector level by using the corresponding sector's price obtained in the previous step. Expressions for deflation of variables are presented in Table 1. Intermediate goods in currency k are deflated by using intermediate goods in constant prices and currency h as:

$$XKR_{ij}^{hk} = XHR_{ij}^{hk} \times \frac{e^{k*}}{e^{h*}} \tag{4}$$

where XKR_{ij}^{hk} is good i in sector j of economy k delivered from economy h in constant prices and currency k and XHR_{ij}^{hk} is good i in sector j of economy k delivered from economy h in constant prices and currency h.

3. The Model Structure

The fundamental structure of the models follows Yano and Kosaka (2008). Sectoral output is determined by the summation of intermediate and final demands, exports to the third world, and statistical discrepancies. Applying a modified model of consumer behavior in Ballard et al. (1985), we endogenize sectoral private consumption among final demand components. In contrast, sectoral price is explained by international price competition of firms. Sectoral output and price are concurrently determined. In this section, using the variables and notations in the Asian International Input-Output Tables in constant prices and national currencies, we describe the structure of a national currency-based international input-output model. Note that we consider the case where international input-output tables have n sectors and r economies.

3.1 Producer Behavior

Consider the following international oligopolistic competition in price:[4]

1) a single firm produces a differentiated good in sector j of economy k,

2) firms in sector j of all economies (i.e., r firms) compete in price within the international market

[4] Due to recent theoretical developments, several multi-country multi-sectoral models apply imperfect competition. See, for example, Swaminathan and Hertel (1996) and Francois (1998).

of good *j*.

Thus, we have *n* international markets in total and there are *r* firms in each market. Under this framework, derived demands and price at the sector level of all economies are explained.[5]

3.1.1 Sectoral Price

In order to produce a differentiated good, a firm employs intermediate goods and labor as inputs. Assume that the firm in sector *j* of economy *k* has the following Cobb-Douglas cost function with economies of scale:

$$C_j^k = \varphi_j^k \left(A_j^k\right)^{-\frac{1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}} \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{\alpha_j^k(L)}{\varphi_j^k}} \prod_{q=1}^{r}\prod_{l=1}^{n}\left[\frac{\left(1+t_l^k\right)P_l^{qk}}{\alpha_{lj}^{qk}(X)}\right]^{\frac{\alpha_{lj}^{qk}(X)}{\varphi_j^k}} \tag{5}$$

where C_j^k is cost function of the firm in sector *j* of economy *k*, A_j^k is an efficiency parameter in production function of the firm in sector *j* of economy *k*, XXR_j^k is output in sector *j* of economy *k* in currency *k*, w_j^k is the wage rate in sector *j* of economy *k*, $\alpha_j^k(L)$ and $\alpha_{lj}^{qk}(X)$ are parameters which satisfies $\varphi_j^k = \alpha_j^k(L) + \sum_{q=1}^{r}\sum_{l=1}^{n}\alpha_{lj}^{qk}(X)$, t_l^k is the tariff rate levied on sector *l* of economy *k*,

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[5] Diewert and Fox (2004) are helpful for the derivations in this subsection.

and $P_l^{qk} = \dfrac{XK_{lj}^{qk}}{XKR_{lj}^{qk}}$.[6] Profit maximization problem of the firm in sector j of economy k is written as:

$$\pi_j^k = P_j^k XXR_j^k\left(\mathbf{P}, P_{SM}^k\right) - C_j^k\left(XXR_j^k\left(\mathbf{P}, P_{SM}^k\right)\right) \tag{6}$$

where $\mathbf{P} = \left(P_i^{hk}\right)$ and P_{SM}^k is price for savings of economy k.[7] The first-order necessary condition for this problem is given by:

$$\frac{\partial \pi_j^k}{\partial P_j^k} = XXR_j^k + P_j^k \frac{\partial XXR_j^k}{\partial P_j^k} - \frac{\partial C_j^k}{\partial XXR_j^k}\frac{\partial XXR_j^k}{\partial P_j^k} = 0 \tag{7}$$

Rearranging equation (7), we obtain the following the inverse elasticity rule:

$$\frac{P_j^k - MC_j^k}{P_j^k} = \frac{1}{\varepsilon_j^k} \tag{8}$$

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[6] Since $\dfrac{XK_{lj}^{qk}}{XKR_{lj}^{qk}} = \dfrac{XH_{ij}^{hk}\left(\dfrac{e^k}{e^h}\right)}{\left(\dfrac{XH_{ij}^{hk}}{P_i^h}\right)\left(\dfrac{e^{k*}}{e^{h*}}\right)} = \dfrac{P_i^h\left(\dfrac{e^k}{e^h}\right)}{\left(\dfrac{e^{k*}}{e^{h*}}\right)}$, input price does not depend on the subscript j.

[7] Details of price for savings are provided in Section 3.2.1.

where

$$
\begin{aligned}
MC_j^k &= \frac{\partial C_j^k}{\partial XXR_j^k} \\
&= \left(A_j^k\right)^{-\frac{1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}-1} \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{\alpha_j^k(L)}{\varphi_j^k}} \prod_{q=1}^{r}\prod_{l=1}^{n}\left[\frac{\left(1+t_l^k\right)P_l^{qk}}{\alpha_{lj}^{qk}(X)}\right]^{\frac{\alpha_{lj}^{qk}(X)}{\varphi_j^k}} \\
&= \frac{1}{\varphi_j^k}\frac{C_j^k}{XXR_j^k}
\end{aligned}
\tag{9}
$$

and

$$
\varepsilon_j^k = -\frac{\partial XXR_j^k}{\partial P_j^k}\frac{P_j^k}{XXR_j^k} \tag{10}
$$

Hence, price in sector *j* of economy *k* is expressed as:

$$
P_j^k = \left(\frac{\varepsilon_j^k}{\varepsilon_j^k - 1}\right) MC_j^k = \mu_j^k MC_j^k \tag{11}
$$

where μ_j^k is the markup factor in sector *j* of economy *k*. As we show the expression, price in sector *i* of economy *h* in currency *k* is explained as:

$$P_i^{hk} = \frac{P_i^h \left(\frac{e^k}{e^h}\right)}{\left(\frac{e^{k*}}{e^{h*}}\right)} \tag{12}$$

3.1.2 Derived Demands

The Shephard's lemma respectively yields intermediate and labor demands as:

$$\begin{aligned} XKR_{ij}^{hk} &= \frac{\partial C_j^k}{\partial\left(1+t_l^k\right)P_{ij}^{hk}} \\ &= \left(A_j^k\right)^{\frac{-1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}} \left[\frac{\alpha_{ij}^{hk}(X)}{\left(1+t_i^k\right)P_i^{hk}}\right] \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{\alpha_j^k(L)}{\varphi_j^k}} \prod_{q=1}^{r} \prod_{l=1}^{n} \left[\frac{\left(1+t_l^k\right)P_l^{qk}}{\alpha_{lj}^{qk}(X)}\right]^{\frac{\alpha_{lj}^{qk}(X)}{\varphi_j^k}} \end{aligned} \tag{13}$$

$$\begin{aligned} L_j^k &= \frac{\partial C_j^k}{\partial w_j^k} \\ &= \left(A_j^k\right)^{\frac{-1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}} \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{-\sum_{i=1}^{n}\alpha_{ij}^k(X)}{\varphi_j^k}} \prod_{q=1}^{r} \prod_{l=1}^{n} \left[\frac{\left(1+t_l^k\right)P_l^{qk}}{\alpha_{lj}^{qk}(X)}\right]^{\frac{\alpha_{lj}^{qk}(X)}{\varphi_j^k}} \end{aligned} \tag{14}$$

Good i delivered from economy h to sector j of economy k in current and constant prices as well as currency h are respectively given by:

$$XH_{ij}^{hk} = XKR_{ij}^{hk} \times P_i^{hk} \times \frac{e^h}{e^k} \tag{15}$$

$$XHR_{ij}^{hk} = \frac{XH_{ij}^{hk}}{P_i^h} \tag{16}$$

3.2 Household Behavior

Household behavior of the model is a slight modification of Ballard et al. (1985). Major dissimilarities are following aspects: 1) our model explains only private consumption: i.e., determination of labor supply is omitted and 2) consumption-savings decision is made by maximizing a Cobb-Douglas utility function, not a constant-elasticity-of-substitution (CES) utility function.[8] In our framework, current consumption and savings are determined in the first stage whereas current consumption is further disaggregated into consumption by sector and economy.

3.2.1 Consumption-Savings Decision

A representative household in economy *k* splits its income into consumption and savings by solving the following utility maximization problem as:

$$\max\ U^k = \overline{CPKR}^{k\,\alpha^k}\ CKR_F^{k\ 1-\alpha^k} \tag{17}$$

[8] In order to prevent from using the saving elasticity for the calibration of parameters in household behavior, we adopt nested Cobb-Douglas utility functions.

subject to

$$YIK^k = \overline{P}^k_{CPK}\overline{CPKR}^k + P^k_{SK}SKR^k \tag{18}$$

where U^k is a Cobb-Douglas utility function of the household in economy *k*, $\overline{CPKR}^k$ is current consumption of economy *k* in constant prices and currency *k*, CKR^k_F is future consumption of economy *k* in constant prices and currency *k*, α^k is a parameter of economy *k*, YIK^k is income of economy *k* in current prices and currency *k*, $\overline{P}^k_{CPK}$ is price for $\overline{CPKR}^k$, P^k_{SK} is price for SKR^k,[9] and SKR^k is savings of economy *k* in constant prices and currency *k*. Current consumption consists of consumption by sector and is expressed as:[10]

$$\overline{CPKR}^k = \prod_{h=1}^{r}\prod_{i=1}^{n}\left(CPKR_i^{hk}\right)^{\lambda^k_{CPK_i^{hk}}} \tag{19}$$

[9] Following Ballard et al. (1985), price for savings of economy *k* is explained by

$P^k_{SK} = \sum_{i=1}^{n} P_i^k \left(\frac{XXR_i^k}{\sum_{l=1}^{n} XXR_l^k}\right)$ where P_i^k is price in sector *i* of economy *k*.

[10] Since current consumption is a Cobb-Douglas composite, we have $\sum_{h=1}^{r}\sum_{i=1}^{n}\lambda^k_{CPK_i^{hk}} = 1$.

where $CPKR_i^{hk} = CPR_i^{hk} \times \left(\frac{e^{k*}}{e^{h*}} \right)$. In order for this problem to be solvable, we establish a linkage between future consumption and savings. By assumption, the representative household purchases capital goods by its savings and lends purchased capital goods to firms. Consequently, the household obtains the expected return per unit of savings which is expressed as $P_K^{Dk} \zeta^k$. In this expression, P_K^{Dk} and ζ^k are price and unit service of capital goods in economy *k*, respectively. Using the return, the household purchases future goods which, by assumption, have the same price as the current consumption of economy *k*, $\overline{P}_{CPK}^k$. As a result, the following equation which bond future consumption to nominal savings of economy *k* is established:

$$P_{SK}^k \; SKR^k = P_{FK}^k CKR_F^k \tag{20}$$

where $P_{FK}^k = \frac{P_S^k \overline{P}_{CPK}^k}{P_K^{Dk} \zeta^k}$. Accordingly, the constraint of the utility maximization problem can be rewritten as:

$$YIK^k = \overline{P}_{CPK}^k \overline{CPKR}^k + P_{FK}^k CKR_F^k \tag{21}$$

Setting up the Lagrangian:

$$L_1^k = \overline{CPKR}^{k^{\alpha^k}} CKR_F^{k^{1-\alpha^k}} + \mu^k \left(YIK^k - \overline{P}_{CPK}^k \overline{CPKR}^k - P_{FK}^k CKR_F^k \right) \tag{22}$$

where μ^k is the Lagrange multiplier of economy *k*, we obtain the following first-order necessary conditions:

$$\alpha^k U^k \overline{CPKR}^{k^{-1}} = \mu^k \overline{P}_{CPK}^k \tag{23}$$

$$\left(1-\alpha^k\right) U^k CKR_F^{k^{-1}} = \mu^k P_{FK}^k \tag{24}$$

$$YIK^k = \overline{P}_{CPK}^k \overline{CPKR}^k + P_{FK}^k CKR_F^k \tag{25}$$

Manipulating these first-order necessary conditions respectively yields current and future consumptions of economy *k* as:

$$\overline{CPKR}^k = \frac{\alpha^k YIK^k}{\overline{P}_{CPK}^k} \tag{26}$$

$$CKR_F^k = \frac{\left(1-\alpha^k\right) YIK^k}{P_{FK}^k} \tag{27}$$

Substituting equation (27) into equation (20) gives savings of economy *k* as:

$$SKR^{k} = \frac{\left(1-\alpha^{k}\right)YIK^{k}}{P_{SK}^{k}} \tag{28}$$

3.2.2 Consumption by Sector and Economy

The household in economy k determines its consumption by sector and economy by solving the following optimization problem:

$$\max \overline{CPKR}^{k} = \prod_{h=1}^{r}\prod_{i=1}^{n}\left(CPKR_{i}^{hk}\right)^{\lambda_{CPK_{i}^{hk}}^{k}} \tag{29}$$

subject to the constraint:[11,12]

$$YIK^{k} - P_{SK}^{k}\,SKR^{k} = \sum_{h=1}^{r}\sum_{i=1}^{n} P_{i}^{hk}\,CPKR_{i}^{hk} \tag{30}$$

The Lagrangian for the second stage utility maximization of the household in economy k can be expressed as:

$$L_{2}^{k} = \prod_{h=1}^{r}\prod_{i=1}^{n}\left(CPKR_{i}^{hk}\right)^{\lambda_{CPK_{i}^{hk}}^{k}} + \psi^{k}\left(YIK^{k} - P_{SK}^{k}\,SKR^{k} - \sum_{h=1}^{r}\sum_{i=1}^{n} P_{i}^{hk}\,CPKR_{i}^{hk}\right) \tag{31}$$

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[11] Due to the principles of double deflation, price for $CPKR_{i}^{hk}$ equals P_{i}^{hk}.

[12] Import duties levied on final demand components are omitted since they do not involve in the determination of sectoral output and price.

where ψ^k is the Lagrange multiplier. Solving this problem, we obtain the following first-order necessary conditions:

$$\lambda^k_{CPK_i^{hk}} \frac{\overline{CPKR}^k}{CPKR_i^{hk}} = \psi^k P_i^{hk}, \qquad h = 1, 2, \ldots r;\ i = 1, 2, \ldots, n \tag{32}$$

$$YIK^k - P^k_{SK} SKR^k = \sum_{h=1}^{r} \sum_{i=1}^{n} P_i^{hk} CPKR_i^{hk} \tag{33}$$

Consequently, consumption in sector *i* of economy *k* delivered from economy *h* in constant prices and currency *k* is given by:

$$CPKR_i^{hk} = \frac{\lambda^k_{CPK_i^{hk}}}{P_i^{hk}} \left(YIK^k - P^k_{SK} SKR^k \right) \tag{34}$$

Substituting equation (34) into equation (29) of this problem yields the price for $\overline{CPKR}^k$.

Using the identity that $YIK^k - P^k_{SK} SKR^k = \overline{P}^k_{CPK} \overline{CPKR}^k$, we have

$$\overline{P}^k_{CPK} = \prod_{h=1}^{r} \prod_{i=1}^{n} \left(\frac{CPKR_i^{hk}}{\lambda^k_{CPK_i^{hk}}} \right)^{\lambda^k_{CPK_i^{hk}}} \tag{35}$$

3.2.3 Household Income and Its Deflator

Wages of economy *k* in currency *k* explains household income of economy *k* in currency *k* as:

$$YIK^k = YIK^k\left(\sum_j w_j^k L_j^k\right) \tag{36}$$

The deflator for household income of economy *k* in currency *k* is determined by the weighted average of sectoral prices of economy *k* (i.e., price for savings of economy *k*) as:

$$P_{YIK}^k = P_{YIK}^k\left(P_{SK}^k\right) \tag{37}$$

3.2.5 Consumption in Currency *h*

Although private consumption determined in this section are denominated in currency *k*, we need them in currency *h* so as to determine sectoral output which is denominated in currency *h*. The conversion can be carried out as:

$$CP_i^{hk} = CPKR_i^{hk} \times P_i^{hk} \times \left(\frac{e^h}{e^k}\right) \tag{38}$$

and

$$CPR_i^{hk} = \frac{CP_i^{hk}}{P_i^h} \tag{39}$$

where CP_i^{hk} is private consumption in sector *i* of economy *k* delivered from economy *h* in current prices and currency *h* and CPR_i^{hk} is private consumption in sector *i* of economy *k* delivered from economy *h* in constant prices and currency *h*.

3.3 Sectoral Output

Sectoral output is determined by summing up the corresponding intermediate and final demands as:

$$\begin{aligned} XXR_i^h &= \sum_{k=1}^{r}\sum_{j=1}^{n} XHR_{ij}^{hk} + \sum_{k=1}^{r} CPR_i^{hk} + \sum_{k=1}^{r} CGR_i^{hk} + \sum_{k=1}^{r} INPR_i^{hk} + \sum_{k=1}^{r} IVR_i^{hk} \\ &+ EXR_i^h + QR_i^h \end{aligned} \tag{40}$$

where XXR_i^h is output in sector *i* of economy *h* in constant prices and currency *h*, CGR_i^{hk} is government consumption in sector *i* of economy *k* delivered from economy *h* in constant prices and currency *h*, INR_i^{hk} is investment in sector *i* of economy *k* delivered from economy *h* in constant prices and currency *h*, IVR_i^{hk} is inventories in sector *i* of economy *k* delivered from economy *h* in constant prices and currency *h*, EXR_i^h is exports to the third world in sector *i* of economy *h* in constant prices and currency *h*, and QR_i^h is statistical discrepancies in sector *i* of economy *h* in constant prices and currency *h*.[13] Note that final demand components (in exception to private consumption) are exogenous in the model.

3.4 Sectoral Wage Rate

Following Yano and Kosaka (2008), the sectoral wage rate is explained by a slight modification of McKibbin and Nguyen (2004, p.47) as:

$$w_j^k = \left(EPC^k\right)^{\beta^k}\left(\frac{XXR_j^k}{L_j^k}\right)^{\xi_j^k} \tag{41}$$

where w_j^k is the wage rate in sector *j* of economy *k*, EPC^k is the expected consumer price of economy *k*, β^k is a parameter of economy *k*, and ξ_j^k is a parameter on sectoral labor productivity in sector *j* of economy *k*.

4. Concluding Remarks

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[13] Note that $EXR_i^h = EGR_i^h + EOR_i^h + EWR_i^h$.

In this paper, we construct national currency-based international input-output tables in constant prices and develop the theoretical structure of a national currency-based international input-output model. Analogous to most multi-country multi-sectoral models, the model has micro foundations: i.e., expressions for producer behavior and household behavior come from profit and utility maximization, respectively. Additionally, the model includes international price competition.

Since the model is one of the global model, we can apply it to analyze global economic issues such as global warming and international trade. Moreover, since the model is national currency-based, it enables us to evaluate national economic policies such as monetary and fiscal policies within a unified international framework. In these respects, the model is quite unique and can provide a new approach to empirical economic analysis. In order to apply the model to economic problems, further work such as estimation and testing of the model is necessary. This is our next research topic.

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Figure 1: The Structure of the 2000 Asian International Input-Output Table

		Intermediate demand (*X*)										Final demand (F)										Export (*E*)				
		I	*M*	*P*	*S*	*T*	*C*	*N*	*K*	*J*	*U*	*I*	*M*	*P*	*S*	*T*	*C*	*N*	*K*	*J*	*U*	*EG*	*EO*	*EW*	*Q*	*XX*
Intermediate input (*X*)	*I*	X^{II}	X^{IM}	X^{IP}	X^{IS}	X^{IT}	X^{IC}	X^{IN}	X^{IK}	X^{IJ}	X^{IU}	F^{II}	F^{IM}	F^{IP}	F^{IS}	F^{IT}	F^{IC}	F^{IN}	F^{IK}	F^{IJ}	F^{IU}	EG^{I}	EO^{I}	EW^{I}	Q^{I}	XX^{I}
	M	X^{MI}	X^{MM}	X^{MP}	X^{MS}	X^{MT}	X^{MC}	X^{MN}	X^{MK}	X^{MJ}	X^{MU}	F^{MI}	F^{MM}	F^{MP}	F^{MS}	F^{MT}	F^{MC}	F^{MN}	F^{MK}	F^{MJ}	F^{MU}	EG^{M}	EO^{M}	EW^{M}	Q^{M}	XX^{M}
	P	X^{PI}	X^{PM}	X^{PP}	X^{PS}	X^{PT}	X^{PC}	X^{PN}	X^{PK}	X^{PJ}	X^{PU}	F^{PI}	F^{PM}	F^{PP}	F^{PS}	F^{PT}	F^{PC}	F^{PN}	F^{PK}	F^{PJ}	F^{PU}	EG^{P}	EO^{P}	EW^{P}	Q^{P}	XX^{P}
	S	X^{SI}	X^{SM}	X^{SP}	X^{SS}	X^{ST}	X^{SC}	X^{SN}	X^{SK}	X^{SJ}	X^{SU}	F^{SI}	F^{SM}	F^{SP}	F^{SS}	F^{ST}	F^{SC}	F^{SN}	F^{SK}	F^{SJ}	F^{SU}	EG^{S}	EO^{S}	EW^{S}	Q^{S}	XX^{S}
	T	X^{TI}	X^{TM}	X^{TP}	X^{TS}	X^{TT}	X^{TC}	X^{TN}	X^{TK}	X^{TJ}	X^{TU}	F^{TI}	F^{TM}	F^{TP}	F^{TS}	F^{TT}	F^{TC}	F^{TN}	F^{TK}	F^{TJ}	F^{TU}	EG^{T}	EO^{T}	EW^{T}	Q^{T}	XX^{T}
	C	X^{CI}	X^{CM}	X^{CP}	X^{CS}	X^{CT}	X^{CC}	X^{CN}	X^{CK}	X^{CJ}	X^{CU}	F^{CI}	F^{CM}	F^{CP}	F^{CS}	F^{CT}	F^{CC}	F^{CN}	F^{CK}	F^{CJ}	F^{CU}	EG^{C}	EO^{C}	EW^{C}	Q^{C}	XX^{C}
	N	X^{NI}	X^{NM}	X^{NP}	X^{NS}	X^{NT}	X^{NC}	X^{NN}	X^{NK}	X^{NJ}	X^{NU}	F^{NI}	F^{NM}	F^{NP}	F^{NS}	F^{NT}	F^{NC}	F^{NN}	F^{NK}	F^{NJ}	F^{NU}	EG^{N}	EO^{N}	EW^{N}	Q^{N}	XX^{N}
	K	X^{KI}	X^{KM}	X^{KP}	X^{KS}	X^{KT}	X^{KC}	X^{KN}	X^{KK}	X^{KJ}	X^{KU}	F^{KI}	F^{KM}	F^{KP}	F^{KS}	F^{KT}	F^{KC}	F^{KN}	F^{KK}	F^{KJ}	F^{KU}	EG^{K}	EO^{K}	EW^{K}	Q^{K}	XX^{K}
	J	X^{JI}	X^{JM}	X^{JP}	X^{JS}	X^{JT}	X^{JC}	X^{JN}	X^{JK}	X^{JJ}	X^{JU}	F^{JI}	F^{JM}	F^{JP}	F^{JS}	F^{JT}	F^{JC}	F^{JN}	F^{JK}	F^{JJ}	F^{JU}	EG^{J}	EO^{J}	EW^{J}	Q^{J}	XX^{J}
	U	X^{UI}	X^{UM}	X^{UP}	X^{US}	X^{UT}	X^{UC}	X^{UN}	X^{UK}	X^{UJ}	X^{UU}	F^{UI}	F^{UM}	F^{UP}	F^{US}	F^{UT}	F^{UC}	F^{UN}	F^{UK}	F^{UJ}	F^{UU}	EG^{U}	EO^{U}	EW^{U}	Q^{U}	XX^{U}
B		BA^{I}	BA^{M}	BA^{P}	BA^{S}	BA^{T}	BA^{C}	BA^{N}	BA^{K}	BA^{J}	BA^{U}	BF^{I}	BF^{M}	BF^{P}	BF^{S}	BF^{T}	BF^{C}	BF^{N}	BF^{K}	BF^{J}	BF^{U}					
G		GA^{I}	GA^{M}	GA^{P}	GA^{S}	GA^{T}	GA^{C}	GA^{N}	GA^{K}	GA^{J}	GA^{U}	GF^{I}	GF^{M}	GF^{P}	GF^{S}	GF^{T}	GF^{C}	GF^{N}	GF^{K}	GF^{J}	GF^{U}					
O		OA^{I}	OA^{M}	OA^{P}	OA^{S}	OA^{T}	OA^{C}	OA^{N}	OA^{K}	OA^{J}	OA^{U}	OF^{I}	OF^{M}	OF^{P}	OF^{S}	OF^{T}	OF^{C}	OF^{N}	OF^{K}	OF^{J}	OF^{U}					
W		WA^{I}	WA^{M}	WA^{P}	WA^{S}	WA^{T}	WA^{C}	WA^{N}	WA^{K}	WA^{J}	WA^{U}	WF^{I}	WF^{M}	WF^{P}	WF^{S}	WF^{T}	WF^{C}	WF^{N}	WF^{K}	WF^{J}	WF^{U}					
D		DA^{I}	DA^{M}	DA^{P}	DA^{S}	DA^{T}	DA^{C}	DA^{N}	DA^{K}	DA^{J}	DA^{U}	DF^{I}	DF^{M}	DF^{P}	DF^{S}	DF^{T}	DF^{C}	DF^{N}	DF^{K}	DF^{J}	DF^{U}					
Value added	*WS*	WS^{I}	WS^{M}	WS^{P}	WS^{S}	WS^{T}	WS^{C}	WS^{N}	WS^{K}	WS^{J}	WS^{U}															
	YC	YC^{I}	YC^{M}	YC^{P}	YC^{S}	YC^{T}	YC^{C}	YC^{N}	YC^{K}	YC^{J}	YC^{U}															
	DP	DP^{I}	DP^{M}	DP^{P}	DP^{S}	DP^{T}	DP^{C}	DP^{N}	DP^{K}	DP^{J}	DP^{U}															
	IT	IT^{I}	IT^{M}	IT^{P}	IT^{S}	IT^{T}	IT^{C}	IT^{N}	IT^{K}	IT^{J}	IT^{U}															
Output		XX^{I}	XX^{M}	XX^{P}	XX^{S}	XX^{T}	XX^{C}	XX^{N}	XX^{K}	XX^{J}	XX^{U}															

Note: *I* is Indonesia, *M* is Malaysia, *P* is the Philippines, *S* is Singapore, *T* is Thailand, *C* is China, *N* is Taiwan, *K* is South Korea, *J* is Japan, *U* is the United States, *EG* is exports to Hong Kong, *EO* is exports to the European Union, *EW* is exports to the rest of the world, *Q* is statistical discrepancies, *B* is international freight and insurance, *G* is imports from Hong Kong, *O* is imports from the European Union, *W* is imports from the rest of the world, *D* is import duties, *WS* is wages and salaries, *YC* is operation surplus, *DP* is depreciations, and *IT* is indirect taxes less subsidies.
Source: Institute of Developing Economies-Japan External Trade Organization (2006b).

Figure 2: Partitions of International Input-Output Tables

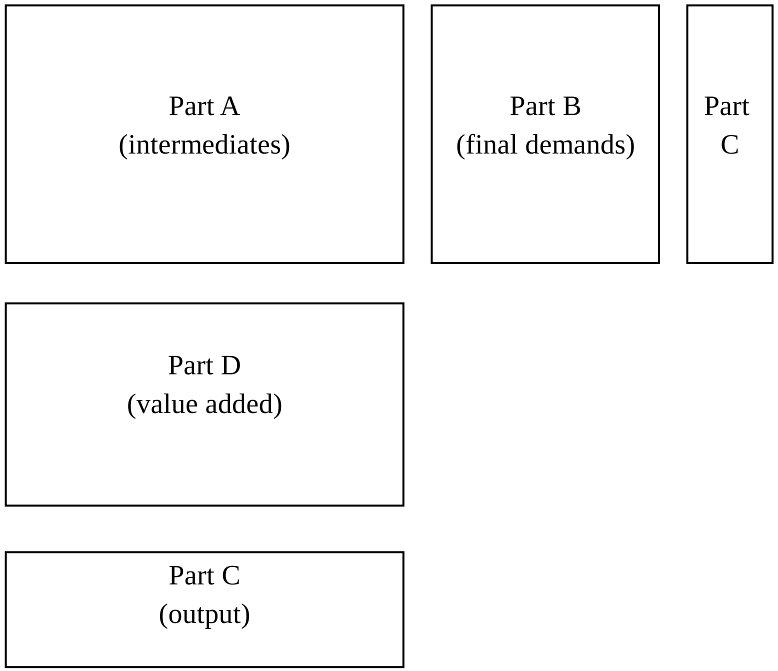

Table 1: Variables of National Currency-Based International Input-Output Tables in Constant Prices

Part	Description	Variables in U.S. Dollars	Variables in Currency h	Variables in Currency k
A	Intermediate good (nominal)	$X\$_{ij}^{hk}$	$XH_{ij}^{hk} = X\$_{ij}^{hk} \times e^h$	$XK_{ij}^{hk} = XH_{ij}^{hk} \times \left(e^k / e^h\right)$
	Intermediate good (real)		$XHR_{ij}^{hk} = XH_{ij}^{hk} / P_i^h$	$XKR_{ij}^{hk} = XHR_{ij}^{hk} \times \left(e^{k*} / e^{h*}\right)$
B	Private consumption (nominal)	$CP\$_i^{hk}$	$CP_i^{hk} = CP\$_i^{hk} \times e^h$	
	Private consumption (real)		$CPR_i^{hk} = CP_i^{hk} / P_i^h$	
	Government consumption (nominal)	$CG\$_i^{hk}$	$CG_i^{hk} = CG\$_i^{hk} \times e^h$	
	Government consumption (real)		$CGR_i^{hk} = CG_i^{hk} / P_i^h$	
	Investment (nominal)	$IN\$_i^{hk}$	$IN_i^{hk} = IN\$_i^{hk} \times e^h$	
	Investment (real)		$INR_i^{hk} = IN_i^{hk} / P_i^h$	
	Inventories (nominal)	$IV\$_i^{hk}$	$IV_i^{hk} = IV\$_i^{hk} \times e^h$	
	Inventories (real)		$IVR_i^{hk} = IV_i^{hk} / P_i^h$	
	Exports to Hong Kong (nominal)	$EG\$_i^h$	$EG_i^h = EG\$_i^h \times e^h$	
	Exports to Hong Kong (real)		$EGR_i^h = EG_i^h / P_i^h$	
	Exports to the European Union (nominal)	$EO\$_i^h$	$EO_i^h = EO\$_i^h \times e^h$	

	Exports to the European Union (real)		$EOR_i^h = EO_i^h / P_i^h$	
	Exports to the ROW (nominal)	$EW\$_i^h$	$EW_i^h = EW\$_i^h \times e^h$	
	Exports to the ROW (real)		$EWR_i^h = EW_i^h / P_i^h$	
	Statistical discrepancies (nominal)	$Q\$_i^h$	$Q_i^h = Q\$_i^h \times e^h$	
	Statistical discrepancies (real)		$QR_i^h = QX_i^h / P_i^h$	
C	Output (nominal)	$XX\$_i^h$	$XX_i^h = XX\$_i^h \times e^h$	
	Output (real)		$XXR_i^h = XX_i^h / P_i^h$	
D	International freight and insurance (nominal)	$BA\$_j^k$		$BA_j^k = BA\$_j^k \times e^k$
	Imports from Hong Kong (nominal)	$GA\$_j^k$		$GA_j^k = GA\$_j^k \times e^k$
	Imports from the European Union (nominal)	$OA\$_j^k$		$OA_j^k = OA\$_j^k \times e^k$
	Imports from the ROW (nominal)	$WA\$_j^k$		$WA_j^k = WA\$_j^k \times e^k$
	Import duties (nominal)	$DA\$_j^k$		$DA_j^k = DA\$_j^k \times e^k$
	Wages (nominal)	$WS\$_j^k$		$WS_j^k = WS\$_j^k \times e^k$
	Operating surplus (nominal)	$YC\$_j^k$		$YC_j^k = YC\$_j^k \times e^k$
	Depreciations (nominal)	$DP\$_j^k$		$DP_j^k = DP\$_j^k \times e^k$

Indirect tax less subsidies (nominal)	$IT\$_j^k$	$IT_j^k = IT\$_j^k \times e^k$

Note: ROW denotes rest of the world, P_i^h is price in sector i of economy h, e^h is the exchange rate of economy h, and e^{h*} is the base-year exchange rate of economy h.

Appendix A: Estimation of Household Income and Savings

Household income is estimated by applying the method in Yano and Kosaka (2008). For reference purposes, this appendix is drawn from Yano and Kosaka (2008).

Although international input-output tables provide wages, wages are not sufficient for income data. Since income of workers other than employees is a fraction of operating surplus in international input-output framework, it is required to add this part to wages. To do so, we estimate a modified version of consumption function in Klein's (1950) Model I which is written as:

$$\sum_{h=1}^{r}\sum_{i=1}^{n} CPKR_i^{hk} = c_1^k \left(\frac{\sum_{j=1}^{n} WS_j^k}{P_{CPK}^k} \right) + c_2^k \left(\frac{\sum_{j=1}^{n} YC_j^k}{P_{CPK}^k} \right) \tag{A1}$$

where WS_j^k is wages in sector *j* of economy *k* in currency *k*, YC_j^k is operating surplus in sector *j* of economy *k* in currency *k*, and $P_{CPK}^k = \sum_{h=1}^{r}\sum_{i=1}^{n} P_i^{hk} \left(\frac{CPKR_i^{hk}}{\sum_{q=1}^{r}\sum_{l=1}^{n} CPKR_l^{qk}} \right)$. Equation (A1) is rewritten as:

$$\sum_{h=1}^{r}\sum_{i=1}^{n} CPKR_i^{hk} = c_1^k \frac{\left[\sum_{j=1}^{n} WS_j^k + \left(\frac{c_2^k}{c_1^k} \right) \sum_{j=1}^{n} YC_j^k \right]}{P_{CPK}^k} \tag{A2}$$

We can interpret the parameter c_1^k and numerator of equation (A2) as the average propensity to consume and nominal income, respectively, Hence, nominal income of economy *k* in currency *k* can be written as:

$$YIK^{k} = \sum_{j=1}^{n} WS_{j}^{k} + \left(\frac{c_{2}^{k}}{c_{1}^{k}}\right)\sum_{j=1}^{n} YC_{j}^{k} \tag{A3}$$

Since savings equal income less total consumption, we can express savings as:

$$SK^{k} = YIK^{k} - \sum_{h=1}^{r}\sum_{i=1}^{n} CPKR_{i}^{hk} \tag{A4}$$

where SK^{k} is savings of economy *k* in current prices and currency *k*.

Appendix B: Derivation of the Cost Function

A firm in sector *j* of economy *k* solves the following cost minimization problem:

$$\min C_{j}^{k} = w_{j}^{k} L_{j}^{k} + \sum_{h=}^{r}\sum_{i=1}^{n} \left(1 + t_{i}^{k}\right) P_{i}^{hk} XKR_{ij}^{hk} \tag{A5}$$

subject to

$$XXR_j^k = A_j^k \left(L_j^k\right)^{\alpha_j^k(L)} \prod_{h=}^{r} \prod_{i=1}^{n} \left(XKR_{ij}^{hk}\right)^{\alpha_{ij}^{hk}(X)} \tag{A6}$$

The Lagrangian can be set up as:

$$J_j^k = w_j^k L_j^k + \sum_{h=}^{r} \sum_{i=1}^{n} \left(1+t_i^k\right) P_i^{hk} XKR_{ij}^{hk} + \lambda_j^k \left(XXR_j^k - A_j^k \left(L_j^k\right)^{\alpha_j^k(L)} \prod_{h=}^{r} \prod_{i=1}^{n} \left(XKR_{ij}^{hk}\right)^{\alpha_{ij}^{hk}(X)} \right) \tag{A7}$$

The first-order necessary conditions are:

$$w_j^k = \lambda_j^k \alpha_j^k(L) \frac{XXR_j^k}{L_j^k} \tag{A8}$$

$$\left(1+t_i^k\right) P_i^{hk} = \lambda_j^k \alpha_{ij}^{hk}(X) \frac{XXR_j^k}{XKR_{ij}^{hk}} \quad (h = 1, 2, \ldots, r;\ i = 1, 2, \ldots, n) \tag{A9}$$

$$XXR_j^k = A_j^k \left(L_j^k\right)^{\alpha_j^k(L)} \prod_{h=}^{r} \prod_{i=1}^{n} \left(XKR_{ij}^{hk}\right)^{\alpha_{ij}^{hk}(X)} \tag{A10}$$

Combining equations (A8) and (A9) yields:

$$\frac{w_j^k}{\left(1+t_i^k\right) P_i^{hk}} = \frac{\alpha_j^k(L)}{\alpha_{ij}^{hk}(X)} \frac{XKR_{ij}^{hk}}{L_j^k} \tag{A11}$$

Manipulating equation (A9), we also have:

$$\frac{\left(1+t_i^k\right)P_i^{hk}}{\left(1+t_l^k\right)P_l^{qk}}=\frac{\alpha_{ij}^{hk}\left(X\right)}{\alpha_{lj}^{qk}\left(X\right)}\frac{XKR_{lj}^{qk}}{XKR_{ij}^{hk}} \tag{A12}$$

Solving equation (A12) for XKR_{ij}^{hk} gives:

$$XKR_{ij}^{k}=\frac{\alpha_{ij}^{hk}\left(X\right)}{\alpha_{lj}^{qk}\left(X\right)}\frac{P_l^{qk}}{P_i^{hk}}XKR_{lj}^{qk} \tag{A13}$$

Substituting equation (A13) into equation (A11) and rearranging the resultant yields:

$$L_j^k=\frac{\alpha_j^k\left(L\right)}{\alpha_{lj}^{qk}\left(X\right)}\frac{\left(1+t_l^k\right)P_{lj}^{qk}}{w_j^k}XKR_{lj}^{qk} \tag{A14}$$

Substituting equations (A13) and (A14) into equation (A10) gives:

$$\begin{aligned}XXR_j^k=A_j^k&\left[\frac{\left(1+t_l^k\right)P_{lj}^{qk}}{\alpha_{lj}^{qk}\left(X\right)}\right]^{\alpha_j^k(L)+\sum_{h=1}^{r}\sum_{i=1}^{n}\alpha_{ij}^{hk}(X)}\left[\frac{\alpha_j^k\left(L\right)}{w_j^k}\right]^{\alpha_j^k(L)}\\&\times\left(XKR_{ij}^{qk}\right)^{\alpha_j^k(L)+\sum_{h=1}^{r}\sum_{i=1}^{n}\alpha_{ij}^{hk}(X)}\prod_{h=1}^{r}\prod_{i=1}^{n}\left[\frac{\alpha_{ij}^{hk}\left(X\right)}{\left(1+t_i^k\right)P_i^{hk}}\right]^{\alpha_{ij}^{hk}(X)}\end{aligned} \tag{A15}$$

Subsequently, the expression for XKR_{lj}^{qk} can be obtained by rearranging equation (A15) as:

$$XKR_{lj}^{k} = \left(A_j^k\right)^{\frac{-1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}} \left[\frac{\alpha_{lj}^{qk}(X)}{\left(1+t_l^k\right)P_l^{qk}}\right] \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{\alpha_j^k(L)}{\varphi_j^k}} \prod_{h=1}^{r}\prod_{i=1}^{n}\left[\frac{\left(1+t_i^k\right)P_i^{hk}}{\alpha_{ij}^{hk}(X)}\right]^{\frac{\alpha_{ij}^{hk}(X)}{\varphi_j^k}} \tag{A16}$$

where $\varphi_j^k = \alpha_j^k(L) + \sum_{h=1}^{r}\sum_{i=1}^{n}\alpha_{ij}^{hk}(X)$. Substituting equation (A16) into (A14) yields the expression for labor demand in sector *j* of economy *k* as:

$$L_j^k = \left(A_j^k\right)^{\frac{-1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}} \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{-\sum_{h=1}^{r}\sum_{i=1}^{n}\alpha_{ij}^{hk}(X)}{\varphi_j^k}} \prod_{h=1}^{r}\prod_{i=1}^{n}\left[\frac{\left(1+t_i^k\right)P_i^{hk}}{\alpha_{ij}^{hk}(X)}\right]^{\frac{\alpha_{lj}^{hk}(X)}{\varphi_j^k}} \tag{A17}$$

Replacing, respectively, the subscripts *h* and *i* with *q* and *l* of equations (A16) and (A17) and substituting the resultant into equation (A5) gives the following cost function:

$$C_j^k = \varphi_j^k \left(A_j^k\right)^{-\frac{1}{\varphi_j^k}} \left(XXR_j^k\right)^{\frac{1}{\varphi_j^k}} \left[\frac{w_j^k}{\alpha_j^k(L)}\right]^{\frac{\alpha_j^k(L)}{\varphi_j^k}} \prod_{q=1}^{r}\prod_{l=1}^{n}\left[\frac{\left(1+t_l^k\right)P_l^{qk}}{\alpha_{lj}^{qk}(X)}\right]^{\frac{\alpha_{lj}^{qk}(X)}{\varphi_j^k}} \tag{A18}$$